HYDRODYNAMICS OF SURFBOARDS

FINAL YEAR THESIS BY MICHAEL PAINE
BACHELOR OF ENGINEERING (MECHANICAL)
UNIVERSITY OF SYDNEY
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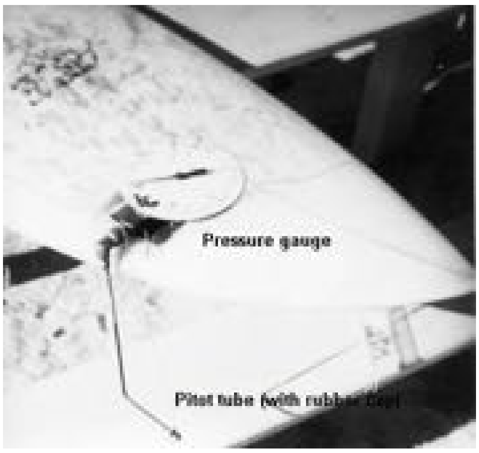
Notes:
1. The thesis was prepared with a typewriter. The attached pages were scanned - my apologises for the poor quality.

2. Shortly after graduating I built a stepped hull surfboard. It worked as intended once planing - it had a very strong resistance to nose-diving- but a serious problem was the extra drag that the step created while paddling. This made it too difficult to catch waves.

Michael Paine 21 May 2001.
https://www.vdrsyd.com/mp/index.html

Scotts Head, NSW 1974

Scotts Head, NSW 1974



Pitot tube on surfboard to estimate velocity

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Mr Halliday,Alec Laws,Arthur,Val,Max,Trevor Shearing, Denise and the anonymous surfers in the photographs.

NOTATION

B....constant in the wave stream function.
b....beam of planing craft.
C_L...lift co-efficient.
C_v...speed co-efficient.
C_Δ...load co-efficient.
C,c..wave velocity (celerity).
D....drag force.
d....water depth below the trough.
F ...Froude number.
g....acceleration due to gravity.
H....wave crest to trough height.
k....cyclic constant in the wave stream function.
k....radius of gyration,in stability equations.
L....characteristic length,in Froude number calculations.
L....lift force.
L_B...bouyant lift.
l....wetted length.
M....moment about the centre of gravity.
p....location of centre of pressure.
r....location of centre of gravity from transom.
S....side force.
u,v..horizontal & vertical velocities,respectively.
v_p...peeling velocity relative to the beach.
v_r...board velocity relative to the water.
v_{orbit}.orbital velocity of the water.
v_w...relative velocity between wave and water.
W....load.
w....water mass density.
x,y..horizontal & vertical co-ordinates.
α....free surface angle with horizontal.
β....wake angle,in free surface plane.
Δ....load.
λ....wavelength,for wave theory.
λ....wetted length to beam ratio.
ρ....density.
ψ....stream function.
τ....trim angle.

Introduction

Although he may not be aware of the fact, there is a simple hydrodynamic explanation why a 'surfer' is able to make a surfboard ride on an ocean wave.

The motion of a surfboard upon a wave is an extreme example of boat/wave interaction and the object of this thesis is to explain, analyse and predict relevant phenomena. Practical research into surfboard velocities and attitudes has been carried out at some of Australia's world renowned surfing beaches and only slightly more mundane experiments conducted with a model surfboard in the Hydrodynamics Laboratory of the University.

Previous Work

It appears that very few scientific investigations have been made into the hydrodynamics of surfboards. To date surfboards have been developed on a trial and error basis. If an exceptionally good design results, then a hydrodynamic explanation may be published in a surfing magazine. Most of these explanations are quite naive but the design peculiarities they describe are often very sound. Surfboards are occasionally mentioned in scientific articles but no analysis has been made of the motion of a surfboard on a wave.

In Australia no information appears to be available on typical speeds of surfboards, although rumours are current of surfers attaching marine speedometers to surfboards, apparently for personal interest only. There is however an abundancy of spectacular photographs showing position and attitude of the surfboard.

The basic principles of surfboard riding.

There are many different styles of surfboard riding but just one common aim; to ride as close as possible to the broken wave, without being caught in the turbulent foam. To achieve this the surfboard rider must find a beach where the waves peel across the beach as they break. He may then ride across the face of the wave, hopefully remaining just ahead of the plunging water. A typical beach plan is shown overleaf and the photograph demonstrates the surfer's trajectory. The profiles correspond to the stations shown on the beach plan. Usually the surfer would be riding a number 3 profile.

A basic knowledge of water wave and planning hull theory is needed for the analysis of surfboard motion. This will be covered in the following sections.

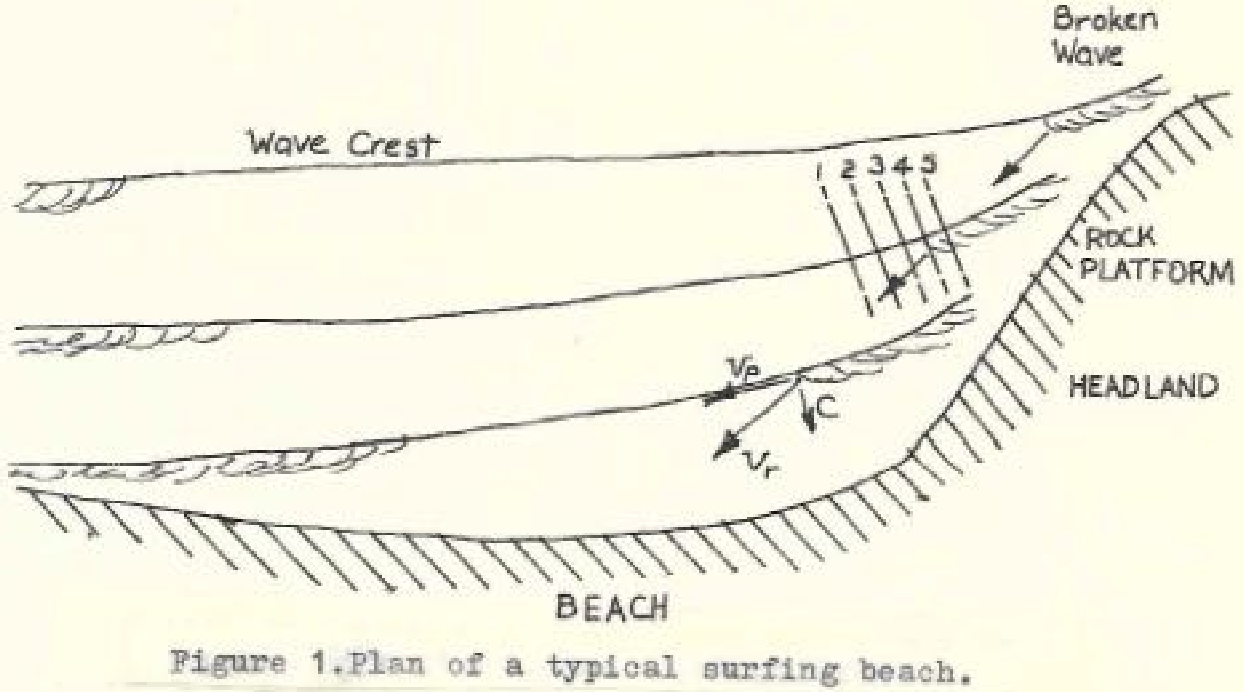


Figure 1.Plan of a typical surfing beach.

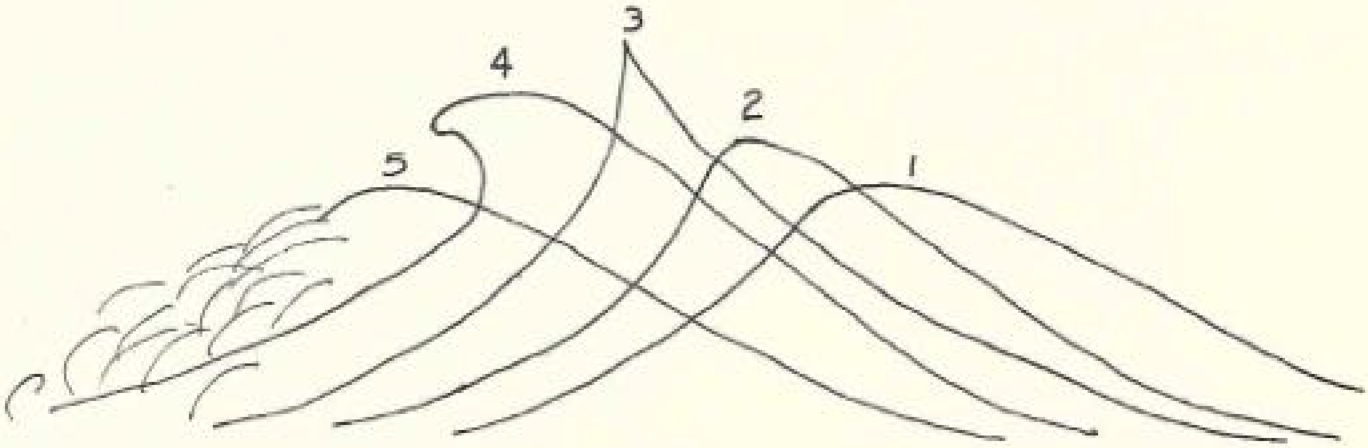


Figure 2.Profiles of a breaking wave
(The numerals refer to the section lines in fig.1)

Ocean waves are generated by winds in mid-ocean. If the wind blows consistantly over a section of ocean, waves will form, their height and wave-length being a function of wind speed and 'fetch' (the length of the relevant section of ocean). The waves propagate in the direction of the wind and may approach the coast. The wave motion changes as the water becomes shallower and may be classified as deep water or shallow water wave motion. Of course, shallow water waves are the main concern of this thesis, however, it is considered beneficial to study deep water waves also.

Deep Water Waves

If the depth of water is greater than the wave-length of the wave train then deep water wave motion occurs. This is characterised by;

1) The water particles perform approximately circular orbits as the wave passes. The profile is trochoidal, the crests are sharper than the troughs.

2) Celerity (wave velocity) is a function of the wave-length.

$$C = \sqrt{g\lambda / 2\pi}$$

Propagation of a deep water wave is shown.

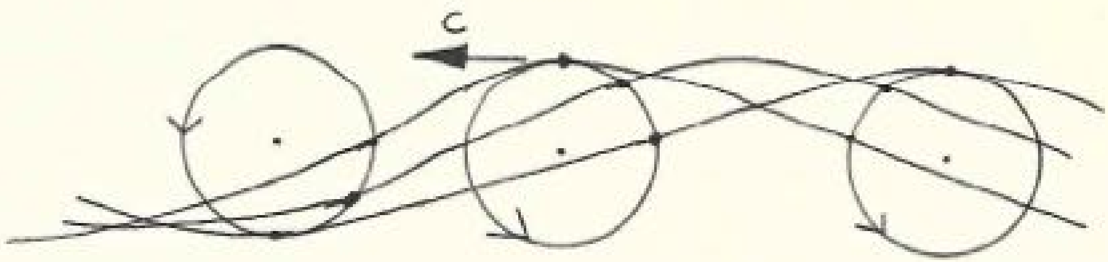


Figure 3. Propagation of trochoidal waves.

The surfboard travels with the wave and the water motion relative to the wave involves a Gallilean transformation in which wave velocity is superimposed on the water particle motion.

To obtain velocities, angles and accelerations at the free surface consider the system as irrotational, then a stream function exists such that;

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

where x,y are horizontal and vertical co-ordinates respectively. By definition of the stream function;-

$$u = -\frac{\partial \psi}{\partial y} \quad \& \quad v = \frac{\partial \psi}{\partial x}$$

where u, v are velocities in x & y directions. Lamb (Ref. A.2) gives the stream function for trochoidal waves as;

$$\psi = c\,(-y + Be^{ky}\cos kx)$$

but at the free surface $\psi = 0$

therefore $y = Be^{ky}\cos kx$ ....(1)

and $u = Ck(\frac{1}{k} - Be^{ky}\cos kx)$....(2)

$v = Ck(-Be^{ky}\sin kx)$ ....(3)

Accelerations are given by,

$$\dot{u} = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$\dot{v} = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

But $\frac{\partial u}{\partial x} = CBk^2e^{ky}\sin kx$ $\qquad \frac{\partial u}{\partial y} = -CBk^2e^{ky}\cos kx$

$\frac{\partial v}{\partial x} = -CBk^2e^{ky}\cos kx$ $\qquad \frac{\partial v}{\partial y} = -CBk^2e^{ky}\sin kx$

therefore

$\dot{u} = CBk^2e^{ky}(u\sin kx - v\cos kx)$ ....(4)

$\dot{v} = -CBk^2e^{ky}(u\cos kx - v\sin kx)$ ....(5)

Also the conditions for continuity and irrotational motion are satisfied.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \& \quad \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$$

The free surface angle is obtained from the derivative,

$$\alpha = \tan^{-1}\left(\frac{dy}{dx}\right)$$

but $y = Be^{ky}\cos kx$

therefore $x = \frac{1}{k}\cos^{-1}(y/Be^{ky})$

$$\frac{dx}{dy} = \frac{-e^{ky}(1-ky)}{k(B^2-y^2e^{-2ky})^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{-k(B^2-y^2e^{-2ky})^{\frac{1}{2}}}{e^{-ky}(1-ky)} \quad \ldots.(6)$$

Equations (1) to (6) have been included in a computer program to aid the analysis. A sample of the output is included as an appendix.

Shallow Water Waves

As the ocean waves propagate into shallow water they slow down and the orbits squash to ellipses. The celerity now depends on the height of the wave and the depth of the water,

$$C \doteq \sqrt{g(d + \frac{H}{2})}$$

where d is depth below trough and H is the crest-trough height. As the waves slow down the wave-length decreases (period remains constant) and this coupled with decreasing depth concentrates the energy of the wave. The wave height increases until the wave becomes unstable and breaks. At this point the orbit velocity of the water has equalled the wave celerity, the wave is translating.

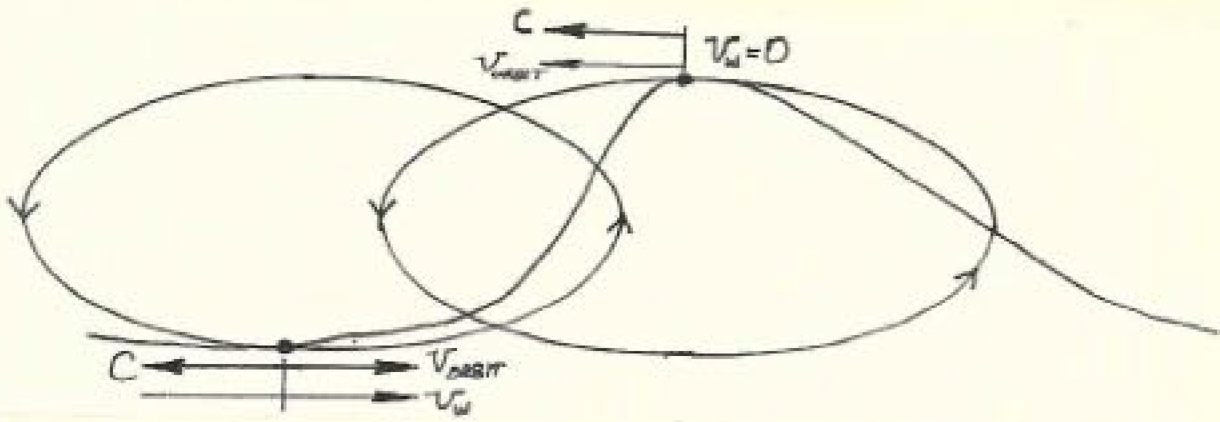


Figure 4. Particle orbits in breaking waves.

The wave/water relative velocities and the free surface angles at various stations on the wave are required. At the crest the relative velocity is zero and in the trough the velocity is approximately 2c. The velocity profile is obtained from;

$$V_W^2 \propto (H-h)$$

where h is the height of the station above the trough.

But at $h = 0, \quad V_W = 2c$

$h = H, \quad V_W = 0$

$\therefore \quad V_W/c = 2\sqrt{1-h/H}$

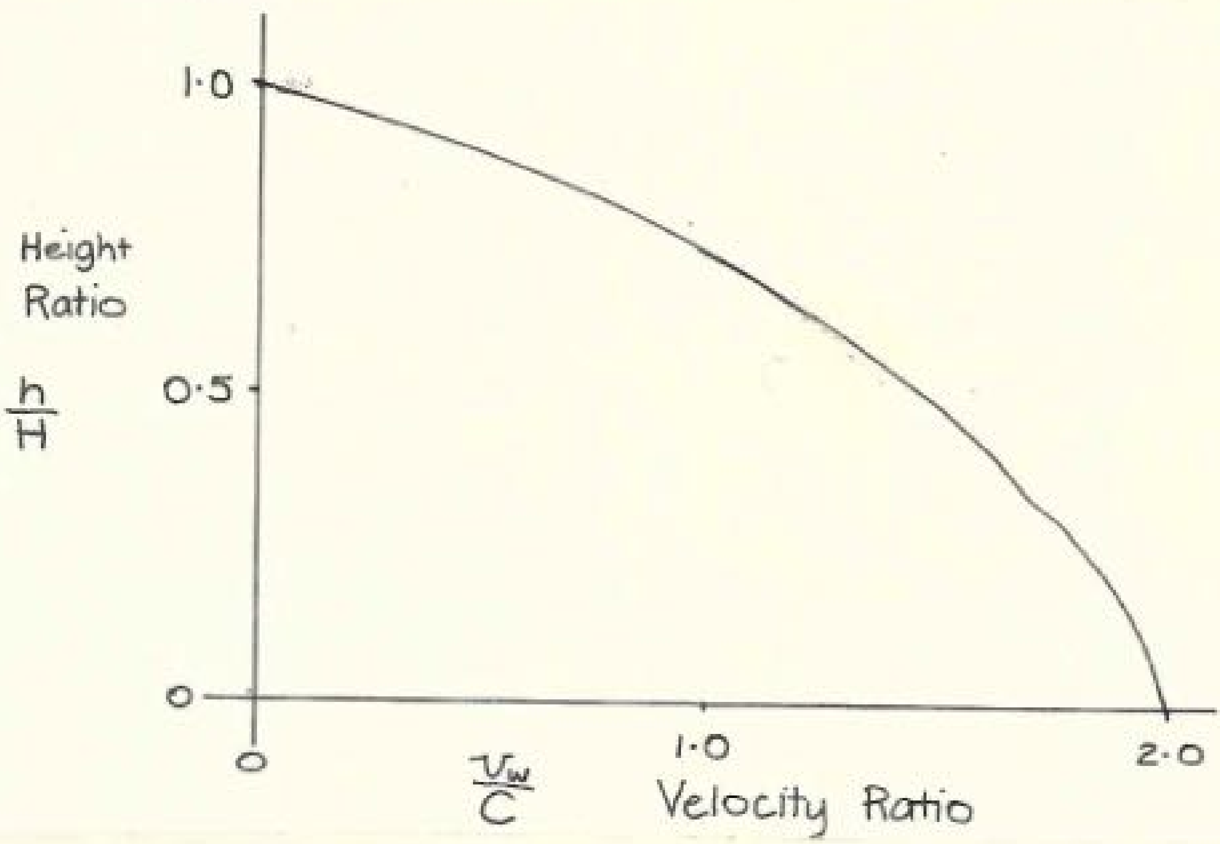


Figure 5. Velocity Profile for a Breaking Wave.

The free surface angles are more difficult to predict. At midheight the orbital velocity is directed vertically and has magnitude C. Therefore, the midheight free surface angle can be expected to be approximately 45° at breaking point. Below midheight it is convenient to use a sinusoidal approximation for the profile and above midheight the profile is unpredictable as shown.

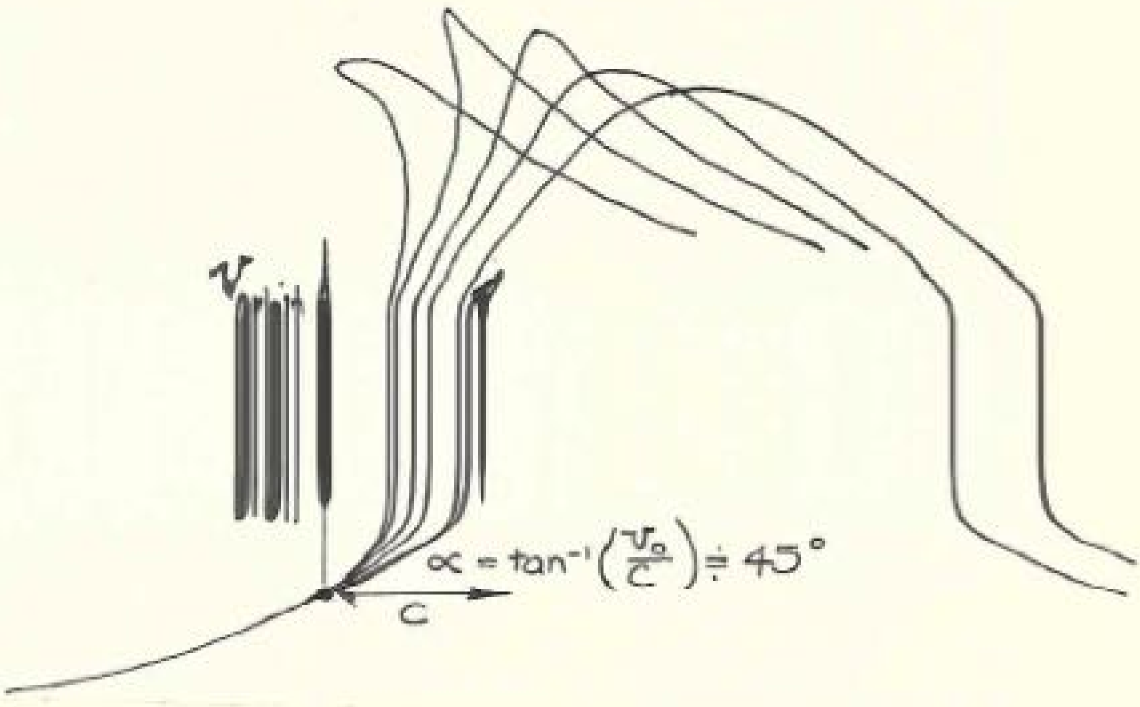


Figure 6. Free Surface Angles for a Breaking Wave.
(Velocities are shown at h= 0.5H)

Typical Waves of Sydney

Typical surfing waves were determined from information in a geography thesis presented in 1964 (Ref. A4). This thesis analysed wave-height, period and direction over a 1½ year period. Relevant results are tabulated below -

Wave Height Range (ft)	0-3	3-7	7-11	11-15	15-19	19+
% occurrence	38	42	14	3	1	2

Waves below 3 feet in height are not of interest to surfboard riders, a typical surfing wave being in the 3-7 feet range and, since waves greater than 7 feet are also ridden, typical waves would be at the higher end of this range. On this basis 6 feet has been selected as the typical Sydney surfing wave height.

Of the 60% waves greater than 3 feet many would be unrideable because they were rough or choppy. Thus the probability of finding a rideable surfing wave at a Sydney beach is rather less than 50%, poor odds for such a popular sport!

Planing Craft Theory

There are two basic types of water craft, namely displacement and planing craft. Displacement craft are characterised by a rounded hull, which moves through the water. Planing craft have sharp edges between the sides and bottom and the stern is cut off suddenly. This type of craft attempts to skim over the water, usually with poor success.

All water-craft may be compared by their speed length ratio, the Froude number

$$F_N = V_r / \sqrt{gL}$$

Displacement craft operate at low Froude number. At high Froude numbers they tend to sink in the water due to the convex bottom. Planing motion is indicated by a rise in the centre of gravity of a craft and planing craft can only truly plane at high Froude numbers, typically $F_n > 1.5$.

Some other co-efficients used in planing craft theory are;

$C_v = V_r/\sqrt{gb}$ speed co-efficient based on the beam.

$C_\Delta = \Delta/wb^3$ load co-efficient.

$C_L = 2C_\Delta/C_v^2$ lift co-efficient.

$= \dfrac{\Delta}{\frac{1}{2}\rho v_r^2 b^2}$

These non-dimensional co-efficients form a basis for comparison of geometrically similar craft. All lengths are usually made non-dimensional by dividing by the beam, giving for example;

$\lambda = l/b$ wetted length-beam ratio.

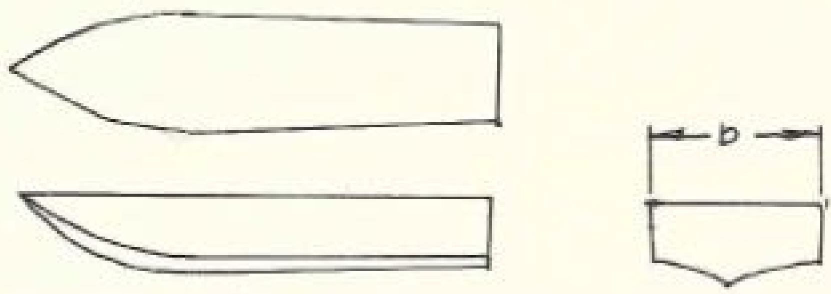


Figure 7. Planing Craft Lines.

Forces on a Planing Craft

The simplest case is a flat-bottomed planing craft. A side view of such a craft is shown;

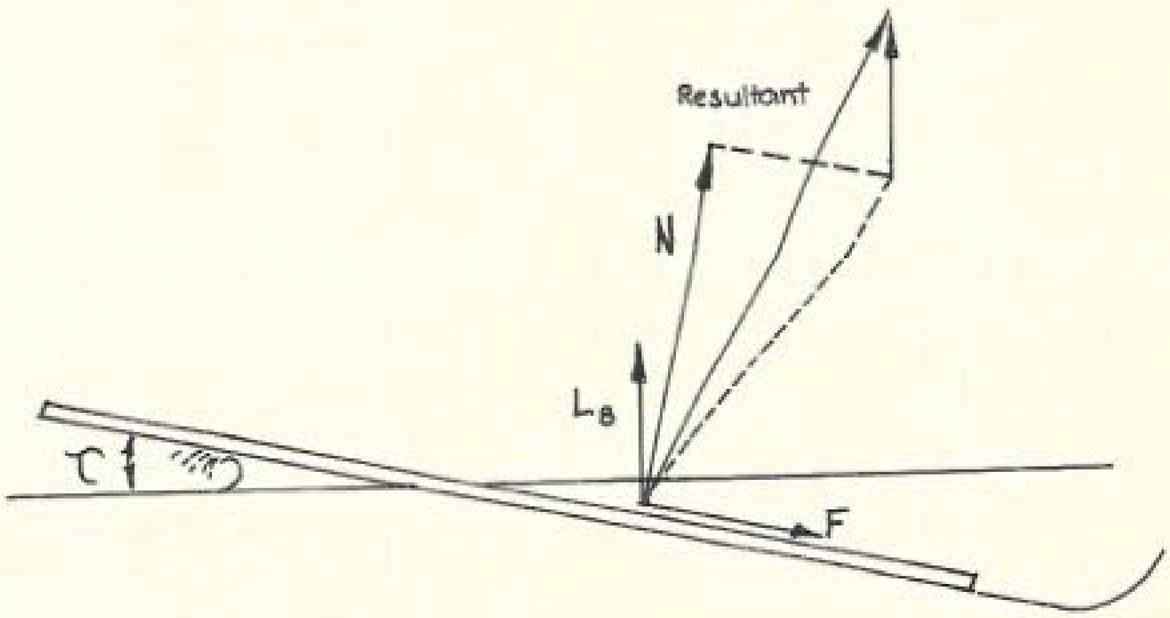


Figure 8. Forces on a Planing Craft.

There are three hydrodynamic forces acting on this craft;

1) Dynamic pressure force due to the change in momentum of the water as it passes under the board. This is resolved normal to the board and is denoted N. This force is the most significant in planing craft, accounting for most of the lift force.

2) 'Skin friction' force due to the flow of water past the surface. This is resolved parallel to the bottom and is denoted F.

3) Hydrostatic pressure force due to the displaced volume of water. Denoted L_B.

Lift is defined as the vertical component of the resultant of these three forces, Drag as the horizontal component. However these definitions assume the free water surface is horizontal. In this thesis a more precise definition is used, namely, Lift is perpendicular to the free surface and Drag is parallel to the free surface.

Ducane (Ref. B.2) gives the following empirical formula known as the E.T.T. formula derived from model tests at the Stevens Institute of Technology.

Lift co-efficient;

$$C_L = \tau^{1.1}(0.012\sqrt{\lambda}+0.0095\lambda^2/C_v^2)$$

Location of the Centre of Pressure;

$$p/l = 0.84\lambda^{-05}/\tau^{0.125}$$

but for surfboard loadings this simplifies to,

$$p/b = \bar{p} = 0.67\lambda$$

Chapter 2: Hydrodynamics of a surfboard on a wave

Initial Theory

Consider first the case of a surfboard travelling straight ahead on a large, deep water wave. This case is useful in understanding the principles of surfboard motion.

The condition for equilibrium is that the surfer travels with the wave maintaining his position on the wave. This could be achieved by the surfer shifting his weight to change the forces on the surfboard. For simplicity assume the free surface is flat in the region of the surfboard, that is, that surface curvature is large compared with the dimensions of the surfboard (The consequences of this assumption are discussed later).

The situation can be represented as shown.

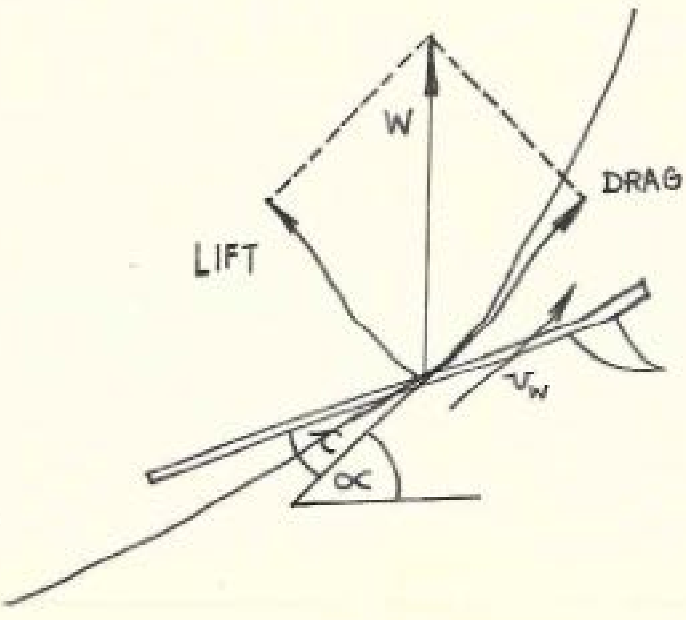


Figure 9. Forces on a Surfboard, Initial Case.

Remembering the definitions of lift and drag being related to the free surface as shown, then for equilibrium;

$$L/D = \cot\alpha$$

$$\text{and} \quad L = W\cos\alpha$$

Therefore if the correct L/D ratio can be maintained the surfboard rider should be able to ride the ocean wave. This is the principle of surfboard riding, manipulation of the forces by changing the trim angle. If the L/D ratio is increased (by stepping forward) the surfer will shoot forward down the face of the wave and if the ratio is decreased by stepping back, the surfer will be swept back over the crest of the wave.

This theory leads to an interesting proposition, the possibility of surfing deep water waves. A surfboard rider may be able to catch and remain on a large ocean wave and travel across sections of ocean with no propulsion unit. To investigate this possibility consider a wave of 300 feet wave-length, its velocity would be about 40 ft/s. The deep water wave computer program gives the following relevant dimensions for these waves.

300ft WAVELENGTH — BOARD; 1·5 ft beam, 200lb Load

WAVE AMPLITUDE H	MAXIMUM FREE SURFACE ANGLE α	$\cot\alpha = \frac{L}{D}$	Relative Velocity V_w		$C_v = \frac{V}{\sqrt{gb}}$	$C_L = \frac{2C_\Delta \cos\alpha}{C_v^2}$	OPTIMUM TRIM (NACA Data)
ft	degrees	—	ft/s		—	$\times 10^2$	degrees
26	16·4°	3·38	34·8		5·00	7·2	18° [1]
21	13·1°	4·30	36·3		5·22	6·7	14° [2]
16	10·1°	5·65	38·0		5·46	6·1	10° [3]
12	7·4°	7·69	38·2		5·50	6·1	6° [4]
7	4·9°	11·75	38·8		5·58	5·9	—

The table also gives L/D ratio required for equilibrium and C_L values for a surfboard with a beam of 1.5 feet and a load of 200 lbs. The optimum trims (trim required for equilibrium) were obtained from a breakdown of NACA test tank data, given as an appendix to this thesis.

Some points of interest are;

1) It is theoretically possible to surf ocean waves of height greater than 16 feet.

2) An initial velocity of 40 ft/s is required to catch the wave. Practically this would be difficult to achieve in such high seas.

3) The sanity of a person riding waves in mid-ocean would have to be questioned.

4) Dynamic instabilities have not been considered in the theory. At the very least the surfer would need constant alertness to ride the wave. Choppy water surfaces would probably lead to the rider becoming exhausted after several hundred yards.

Hydrodynamics of a surfboard travelling across the wave

This is the more common case found in surfboard riding. There is an additional velocity component V_p, defined as the component parallel to the wave crest. Also because the surfboard motion is now asymmetrical there is an additional force component S, the side force, perpendicular to the direction of motion.

The resultant velocity of the surfboard through the water is given by -

$$V_r = \sqrt{V_w^2 + V_p^2}$$ where V_w is the component perpendicular to the wave crest.

Now consider a flat plane tangent to the free surface in the region of the surfboard, the system is as shown.

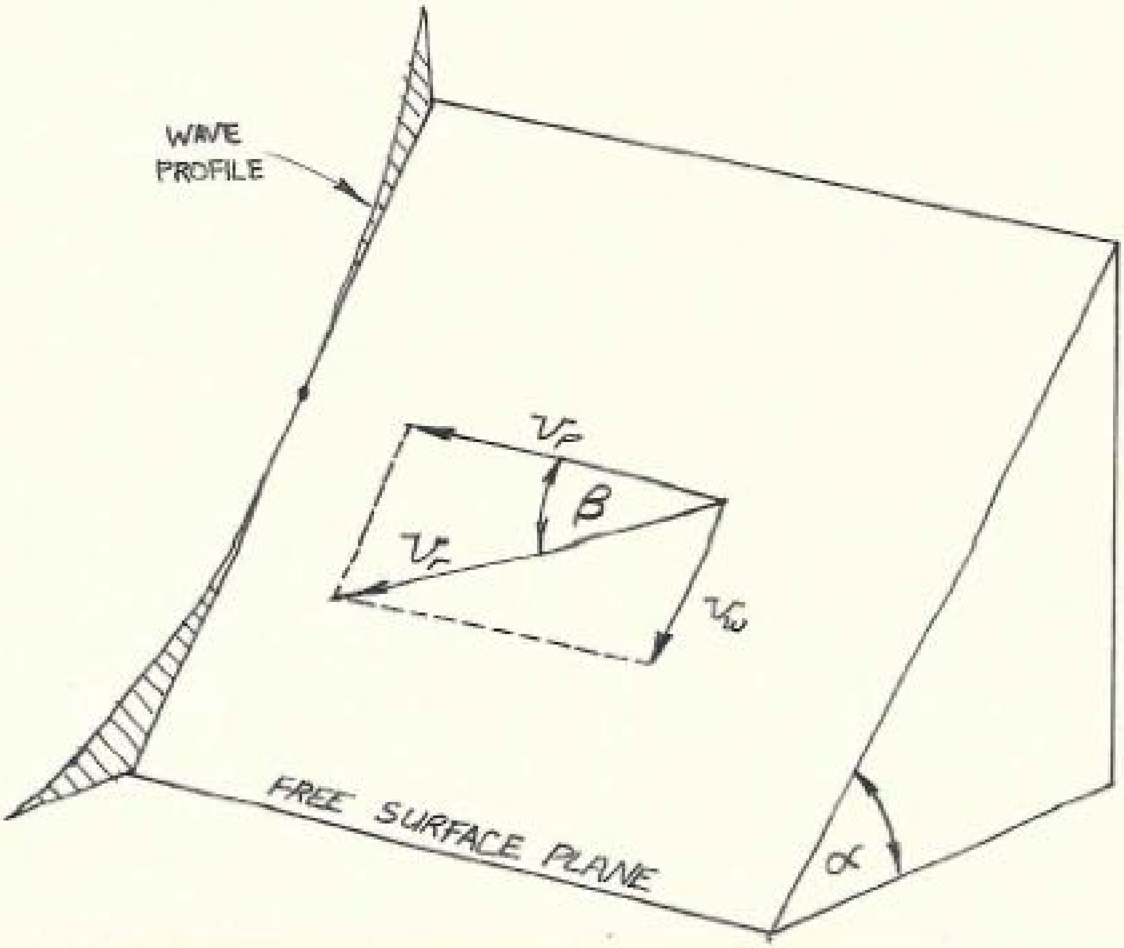


Figure 10. Velocities and Angles.

The angle β is in the free surface plane and is given by -

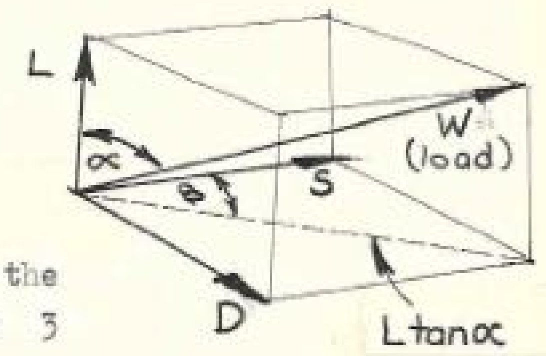
$$\beta = \tan^{-1}(V_w/V_p)$$

For equilibrium the resultant of the hydrodynamic forces exactly opposes the weight of the rider and surfboard. This resultant is resolved into components Lift, Drag and Side force. Defining these;

Lift L - perpendicular to the free surface.

Drag D - in the free surface plan and parallel to the direction of motion through the water.

Side force S - in the free surface plan and perpendicular to the direction of motion.



Vectors S & D are in the free surface plane. Vector L is normal to the free surface.

Resolution of the load vector in 3 directions.

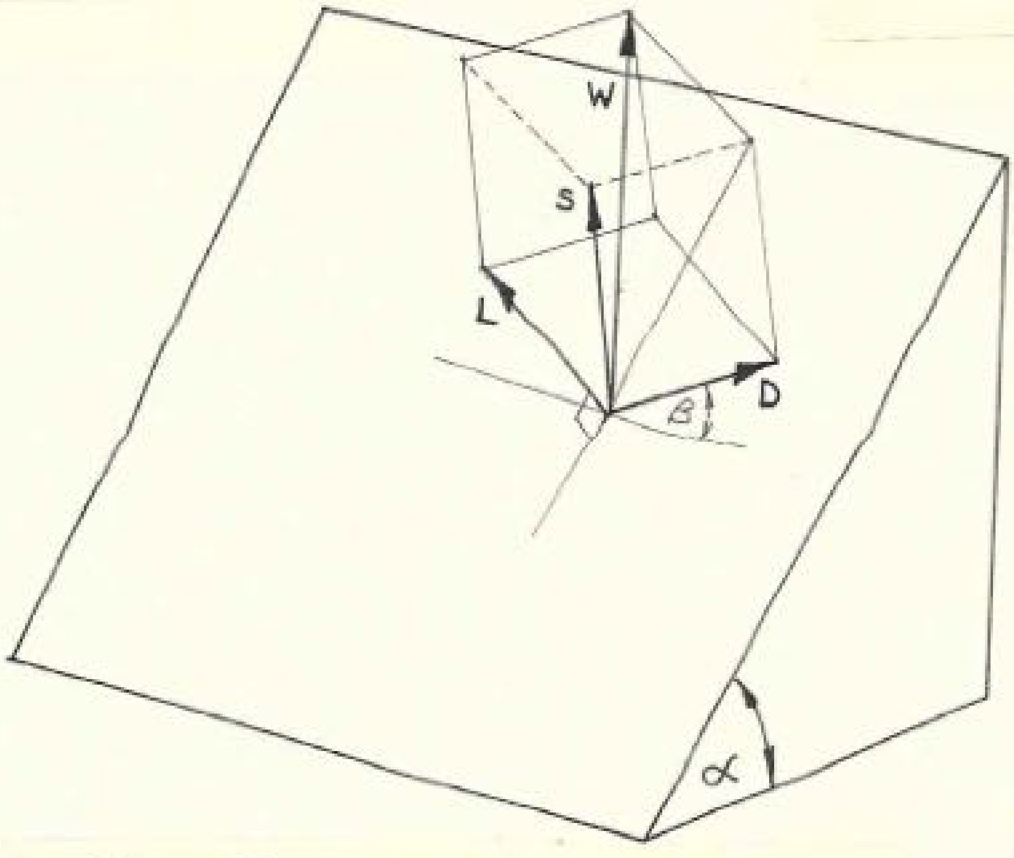


Figure 11. Forces on a surfboard.

Trigonometry gives

$$L = W\cos\alpha$$

$$\frac{L}{D} = \frac{1}{\tan\alpha\sin\beta}$$

and $$\frac{L}{S} = \frac{1}{\tan\alpha\cos\beta}$$

Therefore if the angles α & β are known then the force ratios L/D and L/S can be predicted.

For example let $\alpha = 16^\circ$

$\beta = 45^\circ$ (i.e. $V_w = V_p$)

then for equilibrium motion on the wave (that is no acceleration)

$$\frac{L}{S} = \frac{L}{D} = 5$$

For surfboard dimensions $C_L = 0.1$ and data from NACA TN 4187 gives for the above loading conditions.

Trim = 6°

Yaw = 20°

Roll = 15°

(although these values are not unique, for example yaw may be reduced and roll increased without affecting load ratios.) Verification of these values is difficult since simultaneous velocity and angle measurements are required, however an estimate of the relevant dimensions can be made by analysing a typical photograph.

α is estimated from the surfboard location of the wave.

β is obtained from the direction of the surfboard's wake. This angle is in the free surface plane.

Experiments were carried out at the beach and in the hydrodynamics laboratory. Most of the equipment is new and its design and construction is described. Field work, which was started in May, used imperial units with relevant results converted to metric. The laboratory work used metric units.

Field Experiments

In the absence of available data on typical surfboard velocities, special equipment was manufactured and used to obtain relevant information. The velocities were shown in figure 1.

1) Wave velocity C; The wave velocity perpendicular to the wave crest. At most beaches this is approximately the same as the wave velocity perpendicular to the shoreline.

2) Peeling velocity V_p; The velocity of the surfboard parallel to the wave crest. Again this is approximately the velocity parallel to the shoreline. Peeling velocity is so named because in most cases the surfer travels at the same velocity as the peeling, breaking wave.

3) Surfboard velocity through the Water V_r. The relative velocity between surfboard and the water. This requires a velocity measuring device on the surfboard.

Photographs were taken of typical surfing positions and these have been used throughout the thesis.

Measurement of C and V_p

Method A pair of sighting stations were placed at appropriate locations on the shoreline. They were set with the sights parallel and aimed at the relevant section of the surfboard's trajectory. The surfer's motion across the section was then timed with a stopwatch.

The sighting stations each consisted of a horizontal board pivoted on a tripod stand. The main sights consisted of a pinhole in a bracket and a hairline (copper wire) set across a length of pipe at the other end of the board.

Correct positioning was achieved by means of a small "finder" telescope on the first station being aligned with a pair of graduated scales on the second station. This assembly is shown in figure 12.

Initial alignment of the components was achieved by sighting both stations on some object on the horizon and adjusting the finder telescope to line it up with the scales. Due to parallax error the scales only line up at one point and this is the principle of the alignment method.

The accuracy was estimated by setting the sights on two markers some 50 yards away, the distance between the markers and the stations being the same. The telescope was then used for alignment and the maximum resulting error was less then 1ft. The equipment is accurate to within half a degree.

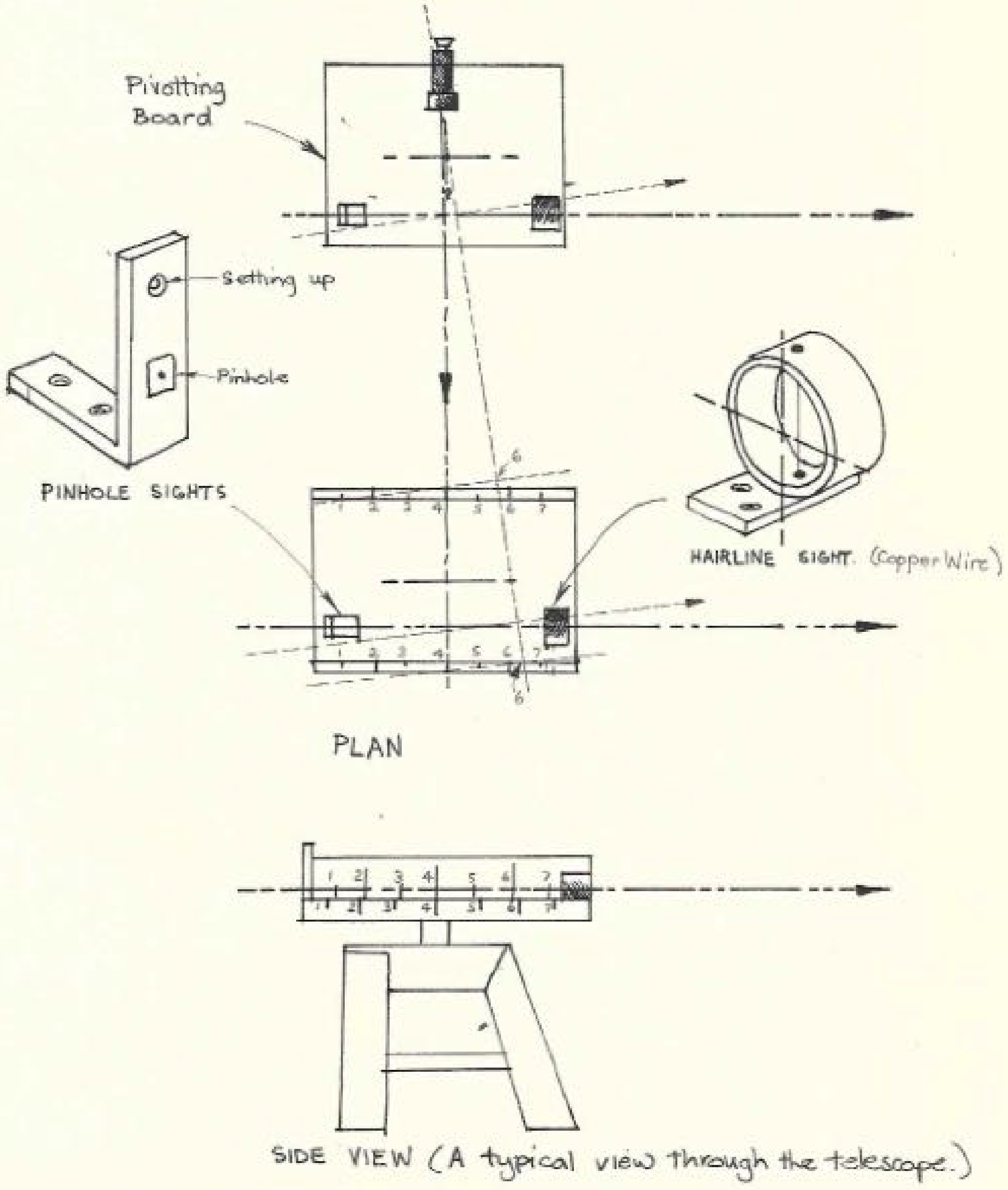


Figure 12. Sighting equipment.

Results

The results of measurements at the beach have been tabulated. It should be noted that peeling velocity is highly variable since it depends on the direction of wave approach.

Board Speed Measurements

Method A velocity measuring device to be attached to the surfboard, the requirements being;

1) Accuracy $\pm$ 10%

2) Reliable and robust

3) Lightweight and small

4) Unaffected by sea-water

Possible devices considered were;

a) Marine speedometer, consisting of a small propeller connected to a calibrated tachometer. These are fairly expensive and not strong enough to withstand some of the violent forces expected.

b) A deflecting pressure plate. Simply a hinged plate and spring that deflects under pressure. This system was not considered accurate or reliable for surfboard application.

c) Pitot tube and pressure gauge to measure the total pressure under the board. This method was selected for the experiments and proved to be reliable and robust. This system was easy to instal on a surfboard and the components were light and simple to calibrate and use.

Pitot tubes

When aligned with the flow, pitot tubes measure the total pressure P_T due to the fluid kinetic and potential energy.

$$P_T = \frac{1}{2} \rho v^2 + P(\text{static})$$

However in these experiments Pstatic is assumed negligible since the depth of immersion of the pitot tube is less then 6 inches. (An error of less than .25 p.s.i.) Also errors due to pressures in the wave are assumed negligible since the first source of error gives an underestimate and the second source gives and overestimate of the total pressure.

No corrections were applied to the readings;

$$\therefore \quad P_{static} \doteq 0$$

$$\therefore \quad V = \sqrt{\frac{2P}{\rho}}$$

and if P is in p.s.i. then for seawater

$$V = 12\sqrt{P} \quad ft/S \quad = \quad 3.66\sqrt{P} \quad m/s$$

Pitot tubes are fairly yaw insensitive. Most authors say 10° is an allowable misalignment.

Equipment

The pitot tube dimensions are shown later in figure 13 . The pressure gauge requirements were;

a) Measurement of low pressures (estimated at 0-20 p.s.i.)

b) Reliable and robust

c) Attitude insensitive

d) Unaffected by sea water.

e) Inexpensive.

The possible gauges were.

1) Manometer tubes, eliminated by requirement (C)

2) Transducers and similar electrical devices are eliminated by (d) and (E)

3) Bellows gauges are very good for low pressures but may be damaged by the extreme pressure occuring in waves.

4) Diaphragm gauge would probably be adequate.

5) Bourdon gauge seems to fill the requirements although accuracy at low pressures is poor. A Bourden gauge was obtained from an industrial merchant and this proved suitable for the task.

The gauge was calibrated with an inclined water filled pipe. The gauge was attached to the pivot point of the pipe and the height of the free surface above the gauge was measured and the head was converted to p.s.i..

Method of attachment of the assembly to the surfboard

Ideally the pitot tube should be located well forward on the board, away from the influence of the boundary layer. A location was selected about 1ft back from the nose, and about 6" deep. The location is shown in the photograph.

The pressure gauge was firmly attached to the surfboard by two screws. A rectangular hole was cut in the deck of the surfboard and a copper plate set into place with fibre-glass and resin. The copperplate has holes and nuts corresponding to holes in the pressure gauge barrel. A sketch of the assembly is shown;

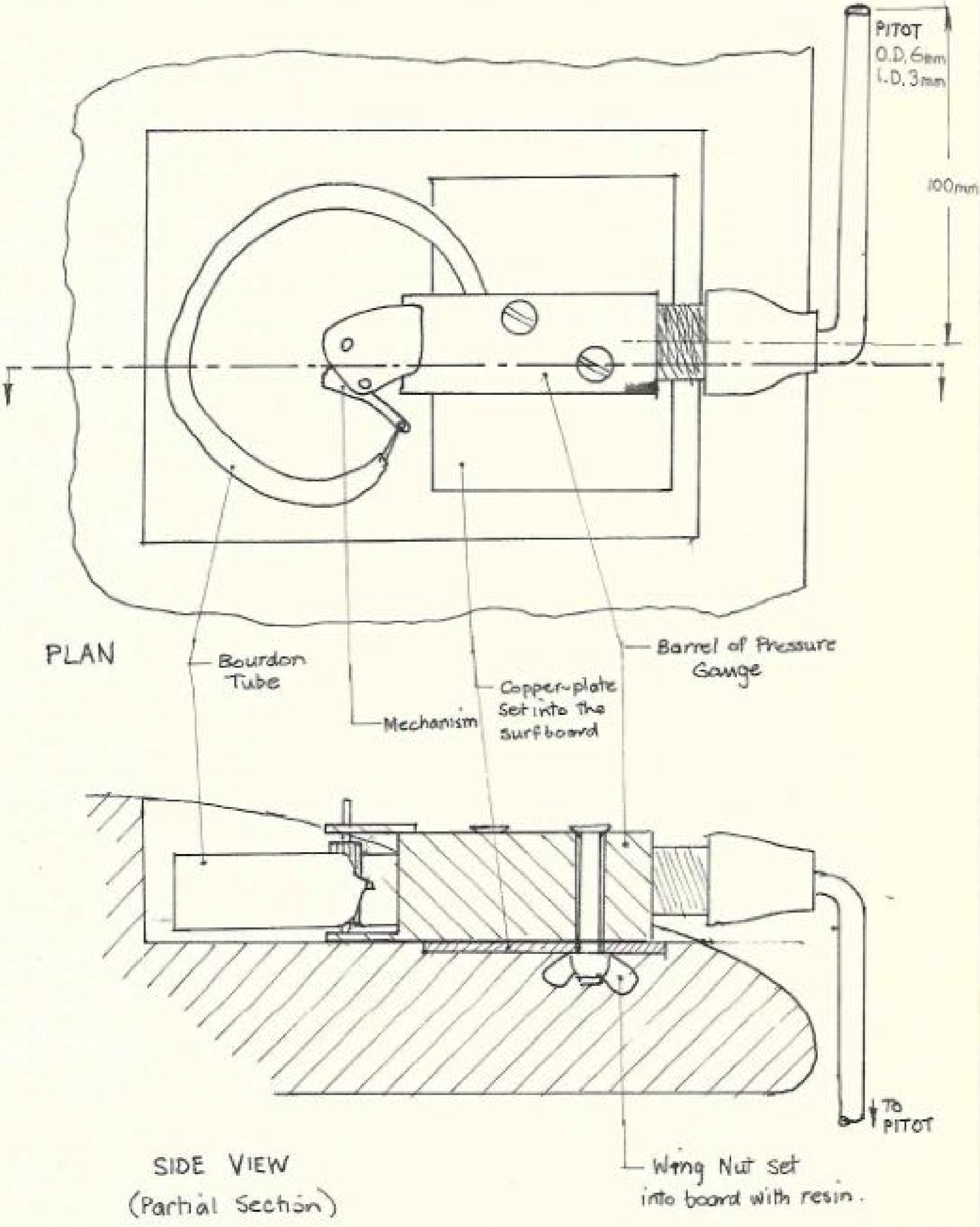


Figure 13. Pressure gauge assembly. (Approx. full size)

- Pitot remained immersed at relevant stages of the surfboard ride.

- Pressure gauge remained high and dry once the surfboard commenced planing.

- The pressure was easy to read.

- The attachment method proved adequate, even when the surfboard was accidentally beached onto the sand.

- The first dial was not stiff enough, it bent when the surfboard was lost in a large wave. A thick, brass plate dial was then substituted and the gauge re-calibrated. This proved satisfactory in service.

Results

The results of board speed measurements were tabulated and also graphs of pressure and velocity against wave height were drawn up.

Discussion

A typical velocity of the surfboard on a 6ft wave was about 24ft/S. The accuracy of this estimate was (based on an 0.5 p.s.i. error in pressure reading) $\pm$ 1.5 ft/S, a 6.25% error. Therefore the results are reliable for an order of magnitude estimate of surfboard velocities.

Some points of interest are;

*There is a linear relationship between wave height and the pressure reading. One explanation is,

$$c = \sqrt{g(d + H/2)}$$

but $d \propto H$

$\therefore$ $c \propto \sqrt{H}$

also $v_r \propto c$

$\therefore$ $v_r \propto \sqrt{H}$

Now for the pressure gauge,

$v_r \propto \sqrt{P}$

$\therefore$ $P \propto H$ ... a linear relationship.

* The wave theory on page 10, combined with the surfboard theory on page 19 gives;

$v_w = 1.5c$ for the surfboard at about mid-height

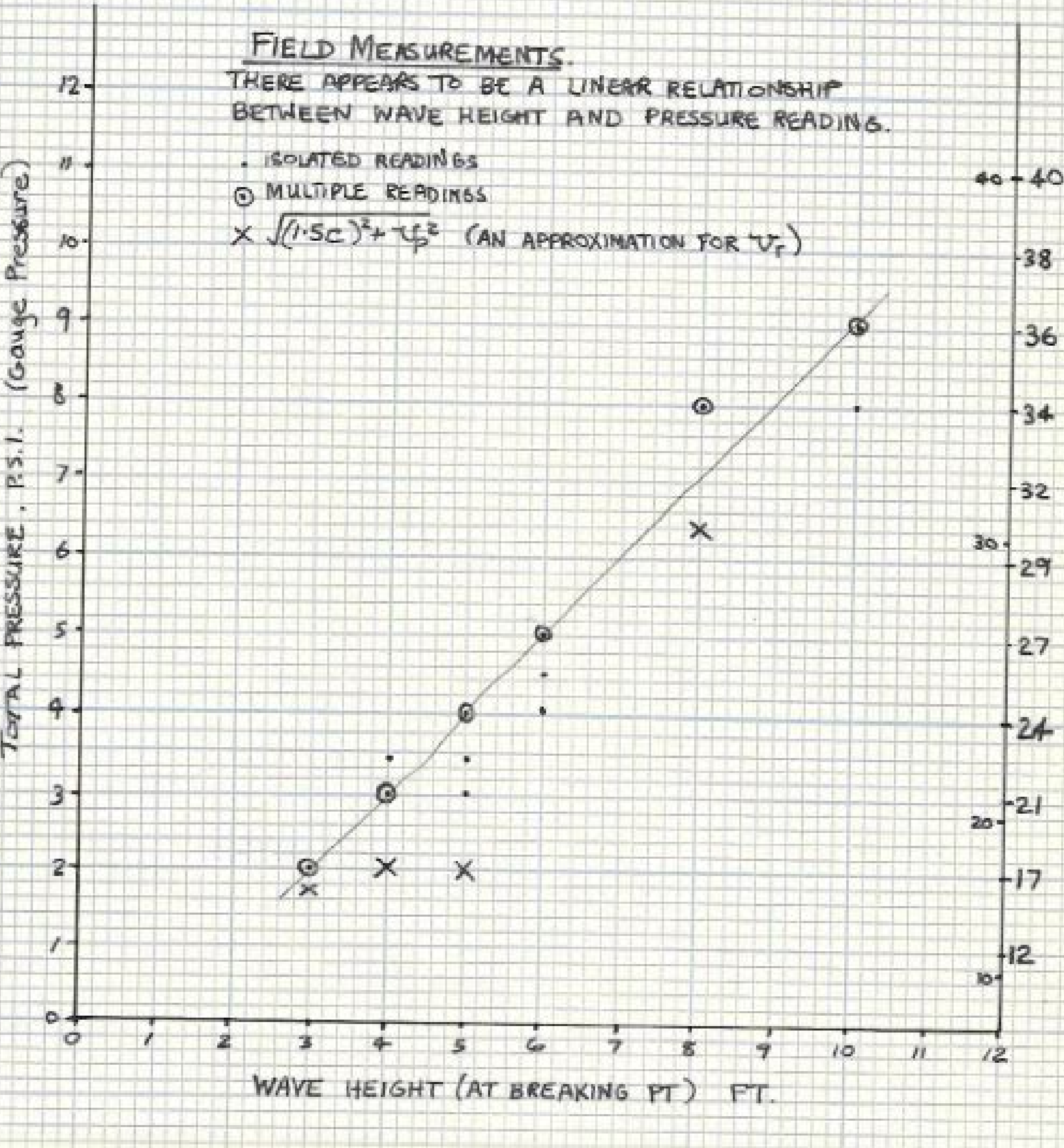
but $v_r = \sqrt{v_W^2 + v_P^2}$

TABLE OF FIELD MEASUREMENTS

Wave Height ft	Pressure Reading psi	V ft/s	V m/s	Comments
				Queenscliff, North End. 11/4/74
				Approx. Wave velocity $c \doteqdot 10$ ft/s $v_p \doteqdot 7$ ft/s
				Morning; Smooth but small waves.
5	3-4	20+	6·8	Shaky first ride, Pitot remained immersed.
5	3·5	22	6·8	Good ride, correct position
6	4	24	7·3	Fast 'left', Pitot still good on left waves.
5	4	24	7·3	Good ride.
				Afternoon; Smaller waves, but occasional
5	4	24	7·3	large wave (One bent the scale of the
4	3·5	22	6·8	pressure gauge when it broke on top of me)
				Queenscliff, North End. 18/7/74
				Approx Wave Velocity $c \doteqdot 10$ ft/s $v_p \doteqdot 5$ ft/s
3	2	16	5·2	Very small, slow waves.
4×3	2	16	5·2	Succession of 3ft waves.
4	2	16	5·2	Slightly larger wave.
				North Steyne 29/7/74
				Wave Velocity $c \doteqdot 10$ ft/s.
				Peeling velocity measured at 7 ft/s (average over 10 waves)
4	3	20	6·3	Slow peeling waves. good position
5	3·5	22	6·8	Good position and trajectory.
5	3·5	22	6·8	" "
5	3	20	6·3	
4	3	20	6·3	
4	3	20	6·3	Waves getting smaller by lunchtime.

TABLE OF FIELD MEASUREMENTS

Wave Height ft	Press. psi	V_r ft/s	V_r m/s	Comments
				Scott's Head (shown in photographs) 25/8/74
				Easy waves, good long ride.
6	5	27	8.2	50 yard ride, ample time to read
6	5	27	8.2	pressure gauge and verify readings.
6	4.5	25	7.8	
6	5	27	8.2	Afternoon readings, waves were smaller,
5	4	24	7.3	Sea breeze made the waves choppy.
5	4	24	7.3	
				Tuesday, 28/8/74. Locals said
10	9	36	11.0	these were the best waves all season.
10	9	36	11.0	(Shown in photographs) Largest waves
10	8	34	10.3	were about 15ft high and the ride
10	9	36	11.0	was about 100 yrds long. The large
8	8	34	10.3	waves were too fast for my board
8	8	34	10.3	so no reading could be taken on them.
8	8	34	10.3	Sea breeze ruined the afternoon waves.
8	8	34	10.3	
	PEELING VELOCITIES			
Distance between stations ft	Time secs.	U_p ft/s	U_p m/s	
				Stations were placed on the beach
48	3.0	16	4.9	and aimed at a typical section
48	2.8	17	5.2	of the surfers ride. Wave heights
42	3.0	14	4.3	were consistant at about 8ft but
42	4.0	10	3.0	the peeling velocity varied with the
42	3.5	12	3.7	angle of wave approach.
42	3.0	14	4.3	
	WAVE VELOCITIES.			
36	2.5	14	4.4	Stations were set up on the
36	2.0	18	5.5	rock platform. Waves were about
36	2.0	18	5.5	8ft high.


FIELD MEASUREMENTS.
THERE APPEARS TO BE A LINEAR RELATIONSHIP BETWEEN WAVE HEIGHT AND PRESSURE READING.
. ISOLATED READINGS
⊙ MULTIPLE READINGS
× $\sqrt{(1.5c)^2 + v_p^2}$ (AN APPROXIMATION FOR v_r)
TOTAL PRESSURE, P.S.I. (Gauge Pressure)
VELOCITY (NON-LINEAR SCALE) FT/S
WAVE HEIGHT (AT BREAKING PT) FT.
0 1 2 3 4 5 6 7 8 9 10 11 12
12 17 21 24 27 29 32 34 36 38 40
10 20 30 40

Aim Initially the experimental work was to have investigated unsymmetrical planing conditions as found in surfboard riding. However research revealed an ample amount of experimental data (notably NACA TN 4187). While these results did not cover the entire range of attitudes found in surfing they made any further work seem futile, considering the equipment and time that went into the NACA experiments.

After some discussion with Mr. Halliday it was decided that a different experimental programme should be attempted, namely creating a standing wave in the flume(circulating water tank) and investigating the properties of a model surf-board on the wave.

Creating a Standing Wave

The main reference for this topic is by Bakhmeteff "Hydraulics of Open Channels" (Ref. C1) On page 61 he recollected an experiment performed in 1911 on the creation of a standing wave in a flume. Luckily he explained the equipment used and even supplied a dimensioned sketch of his standing wave. He also mentions "playing" with model ships on the wave.

The method of creating a standing wave is to produce supercritical flow with a sluice gate and place a small barrier in this flow. At low flow rates a hydraulic jump will form and submerge the flow from under the sluice gate. When the head behind the sluice gate is increased the flow increases and the hydraulic jump may be swept down stream, leaving a standing wave over the barrier. The entire layout is sketched.

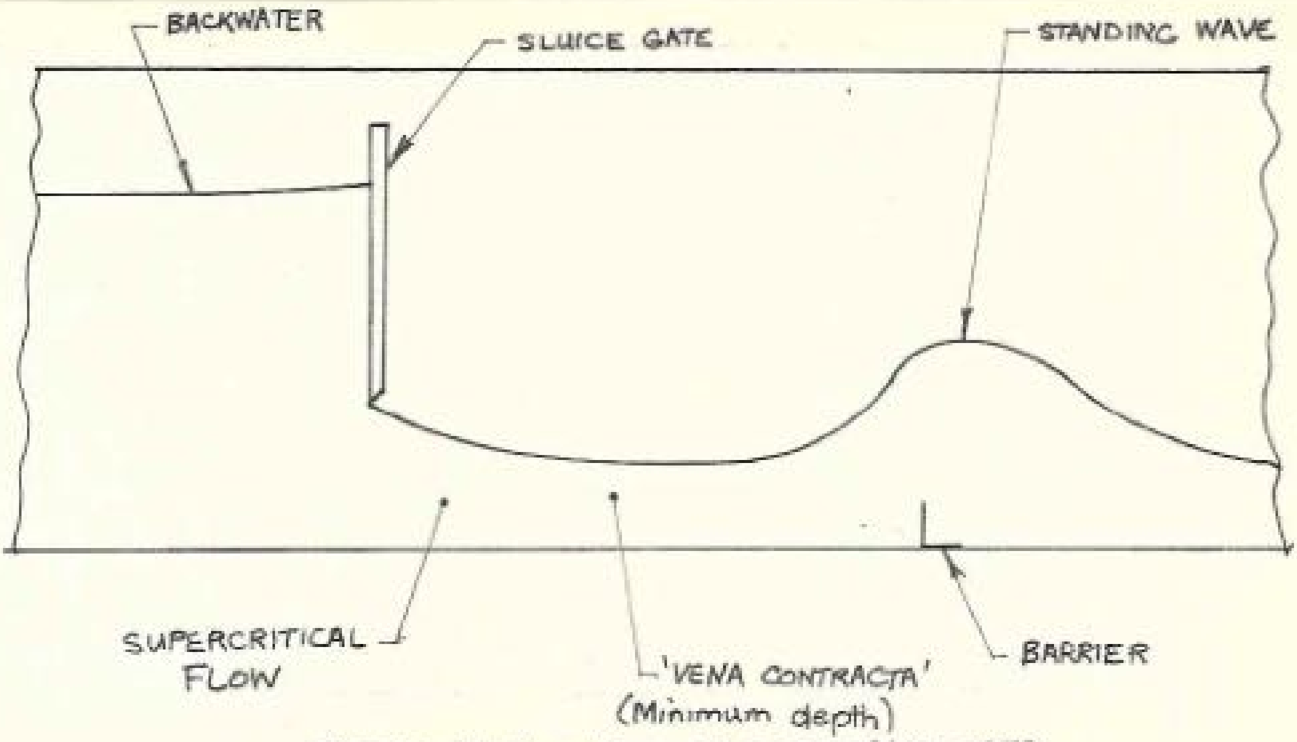


Figure 14.Creation of a standing wave.

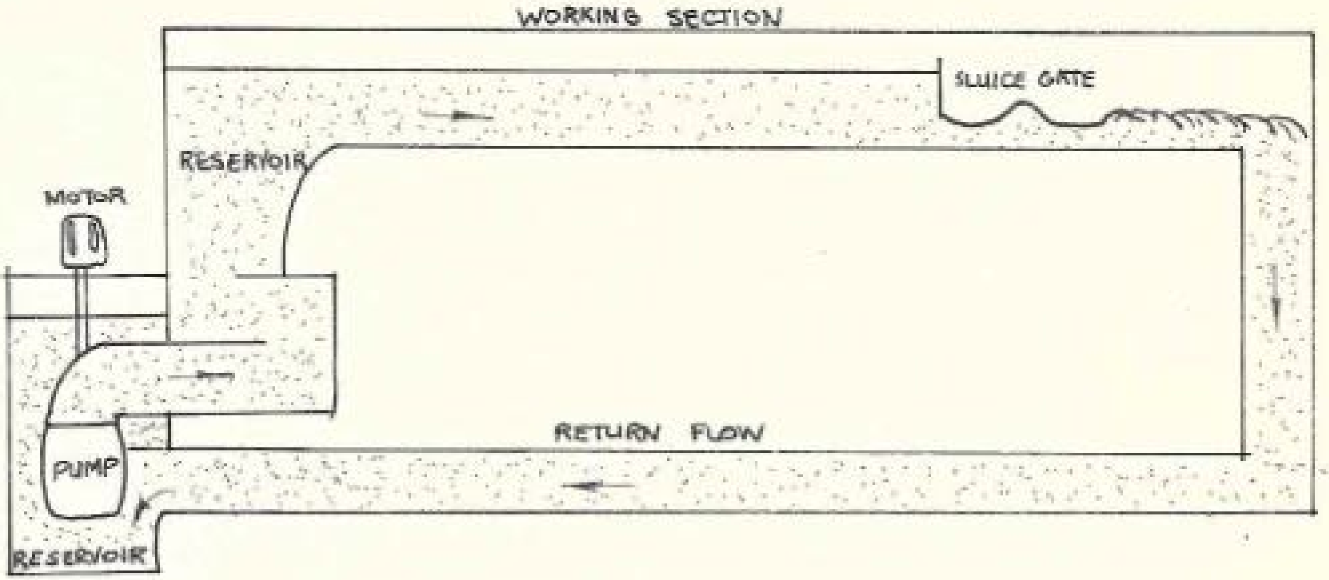


Figure 15.Sydney University circulating water tank.

Model Wave Theory

Comparison of Model Waves with Sea Waves

This theory draws heavily on an article by Canfield and Street (Reference C3) "shoaling of solitary waves on small slopes". The authors compare model solitary waves created for their experiments with large ocean waves breaking on the beach. Quoting directly; "For practical purposes dispersive waves in their final stages of run-up on a beach may be indistinguishable from cnoidal or solitary waves. Therefore in studying large waves it is reasonable to begin in the laboratory by generating and studying non-dispersive (i.e. solitary) waves".

Usually solitary waves are generatedin a towing tank by longitudinal displacement of a barrier. The problem here is to prove that Bakhmeteff's standing wave is a close approximation to a solitary wave. The evidence is unscientific,

a) The water particle motions (relative to the wave) are similar. The streamlines contain an approximately versine hump.

b) Celerities are similar. This comparison is based on Bakhmeteffs experimental results and the theoretical celerity of an equivalent solitary wave.

d) The method of generation is similar, by displacement of water rather than by reaction at the free surface, the case with wind generated waves.

The experimental work is based on the assumption that the standing wave is a close approximation to the solitary wave. This in turn is a good model for shoaling ocean waves.

Criteria for applying model wave results to sea waves

1) Viscous forces and turbulence.

Comparing the Reynolds number of the model and ocean waves.

Let

$$R_N = \frac{UD}{\nu} \quad \text{where}$$

U is initial water velocity relative the bottom.

D is the depth of water initially

ν is the kinematic viscosity

Then for an ocean wave

$$U = 10 \text{ft/s}$$

$$D = 5 \text{ft}$$

$$\nu = 1.2 \times 10^{-5} \ \text{ft}^2/\text{see}$$

$$\therefore \quad \left[Rn\right]_o = 4.17 \times 10^6$$

For the model wave

$$U = 10 \text{ft/s} \quad (3 \text{ m/s})$$

$$D = 0.25 \text{ ft} \quad (80\text{mm})$$

$$\therefore \quad \left[Rn\right]_m = 2.08 \times 10^5$$

For channel flow this Reynolds number range is insignificant however the difference may be significant in model surfboard similitude.

2) Froude number similitude

The existence of a free surface implies that froude similitude is relevant in the comparison.

$$F_n = \frac{C}{\sqrt{gD}} \quad \text{but } C = \sqrt{g\left(\frac{D+H}{2}\right)}$$

$$\therefore \quad F_n = \frac{\sqrt{g\,\frac{(D+H)}{2}}}{\sqrt{gD}} = \sqrt{\frac{1+H/D}{2}}$$

Therefore, providing the $\frac{H}{D}$ ratio is maintained then Froude similitude is satisfied.

3) Critical Flow of the Water

For supercritical flow the depth is less than the critical depth $D\ crit = \frac{U^2}{g}$

from the previous values;

$$[D]_m < [D\ crit]\ \text{model} = (6)^2/32.2 = 1.11\ \text{ft} = 340\text{mm}$$

$$[D]_o \doteq [D\ crit]\ \text{ocean} = (10)^2/32.2 = 3.10\text{ft}$$

This comparison is not significant because the ocean case is not channel flow. Even so the ocean water is close to supercritical flow. This prediction is supported by the observation that both volumes of water contain a large amount of energy for their depth, a characteristic of supercritical flow.

Design of the Equipment

The circulating water tank in the Hydrodynamics laboratory at Sydney University has the following capacities and dimensions.

Glass Working Section length; 2.74 m (9ft)

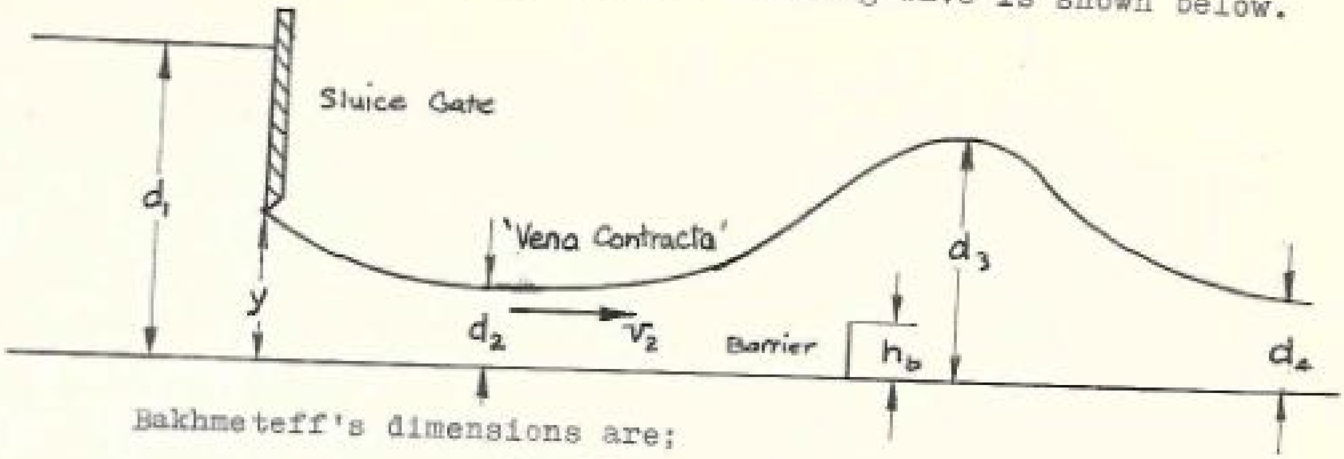
Glass Working Section width; 1.22m (4ft)

Maximum flow rate; .37 cumecs (13 cusecs)

(0.30 cumecs/metre width)

The equipment was designed according to Bakhmeteff's dimensioned drawing.

The equipment for creating a standing wave is shown below.



Bakhmeteff's dimensions are;

$d_2 = 75mm$

$d_3 = 204mm$

$d_4 = 87mm$

$h_b = 40mm$

$q = 14.2$ litres/sec/10cm,width

$\therefore Q = 0.173$ cumecs for the flume.

$v_2 = 1.9$ m/s

$\therefore F_N = 1.9/\sqrt{9.8x0.075} = 2.2$

(Transitional jump conditions,according to Sellin)

For the experiment;

Designing for maximum capacity of the flume,

$Q = 0.368$ cumecs

$\therefore q = 0.3$ cumecs/metre width

now $v_2 = q/d_2$ 1

but we require $F_N = 2.2$,and by definition of F_N

$v_2 = 2.2\sqrt{gxd_2}$ 2

1 &2 give,

$q = 2.2\sqrt{gxd_2^3}$

therefore $d_2 = ((q/2.2)^2/g)^{\frac{1}{3}} = 124mm$

and $v_2 = 0.3/0.124 = 2.4$ m/s

For the sluice;

$d_2 = C_c y$ (C_c is the contraction co-efficient)

Sellin states $C_c = 0.61$

$\therefore y = d_2/C_c = 200mm$

Also from Sellin,

$q = C_D y\sqrt{2gd_1}$

but $C_D = 0.5$ for $d_1/y \doteq 2$

therefore $d_1 = (0.3/0.5x0.2)^2/2g = 460mm$

checking, $d_1/y = 2.3$ therefore the C_D value is correct.

Critical depth of the flow is;

$d_c = \frac{2}{3}(v_2^2/2g + d_2) = 280mm$

Expected dimensions of the equipment are;

$d_1 = 460mm$

$d_2 = 124mm$

$d_3 = 300mm$

$d_4 = 150mm$

$y = 200mm$

$h_b = 40mm$

$Q = 0.37$ cumecs (full capacity)

$v_2 = 2.4$ m/s

Force on the sluice gate F_g

$$F_g = w(d_1 - d_2)^3 / 2(d_1 + d_2)$$
$$= 0.032 \text{ kgf/mm width}$$
$$= 400 \text{ N total}$$

The sluice gate needed to be adjustable. To achieve this the pivot was placed upstream as shown. The moment of F_g about the pivot opposes the weight of the assembly, making adjustment simple during operation of the flume.

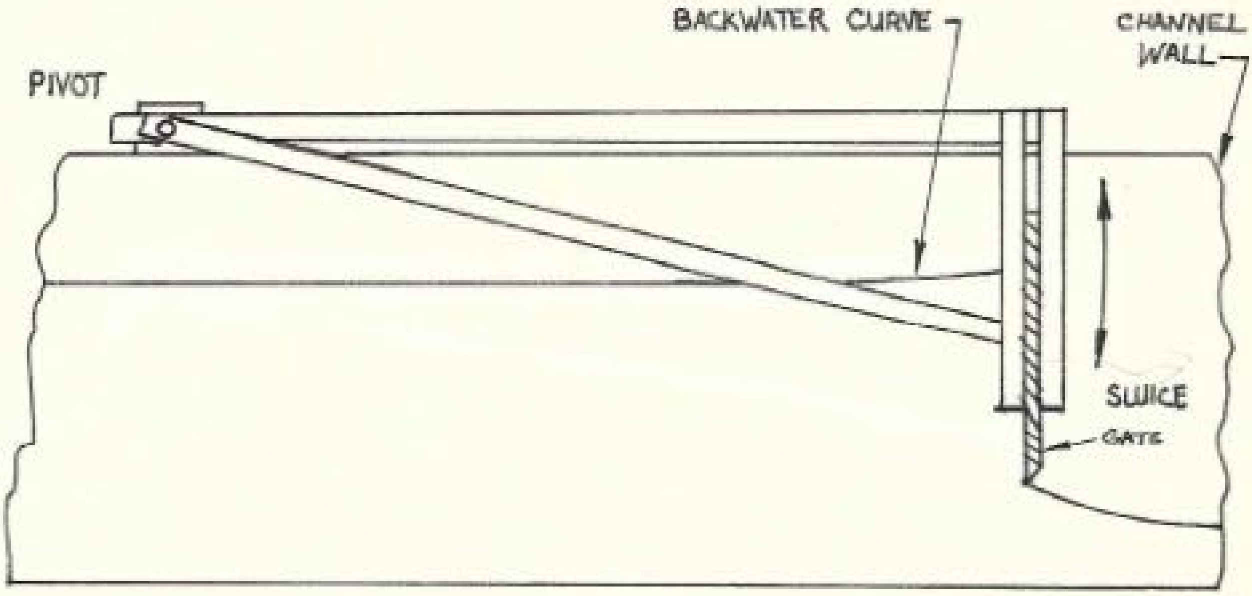


Figure 15. Sluice gate adjustment assembly.

Performance of the Equipment

The sluice gate and barrier were set up in the circulating water tank and a standing wave was produced with only minor adjustments to the sluice gate and barrier. The wave was close to the predicted size so it was assumed that the sluice gate theory was sound and could be used to estimate velocities of the flow.

Model Experiments

Aim The original intention was to place a model surf-board on the standing wave and adjust the load to obtain zero drag or thrust. This was intended to represent a surfer shifting his weight to produce an equilibrium motion on the wave. The trim angle and load position could then be measured and compared with theoretical predictions. Equipment was built for this purpose, but some unforseen problems with stability led to the failure of this experiment.

Equipment

The drag balance consisted of a simple pendulum as shown in figure 17. It was a zero reading apparatus designed to indicate equilibrium conditions.

The surfboard models were initially wooden but later aluminum models were used to give correct values of moment of inertia. The models were all very simple, they had flat bottoms, a small fin and a turned up nose (bow).

Model Surfboard Design

Ducane (Ref. B2) proposes a linear proportionality between wave and ship and wave and model, therefore;

$$\frac{\delta m}{Hm} = \frac{\delta s}{Ho}$$

m - model
s - ship
o - ocean

But δs = 500 mm

Ho = 1800 mm

Hm = 180 mm

$\therefore$ δm = 50 mm

i.e. the model is 1/10th full size.

Froude similitude

We require $[Cv]\ (m) = [Cv]\ (s)$

where $Cv = V/\sqrt{gb}$

$[Cv]\ (m) = 2000/\sqrt{9800 \times 50} = 2.85$

$[Cv]\ (s) = 7000/\sqrt{9800 \times 500} = 3.20$

Therefore Froude similitude is satisfied.

Loading Co-efficient C_Δ

$$C_\Delta = \frac{\Delta}{wb^3}$$

For a surfboard $\Delta = 90$ kg

$b = 500$ mm

$W = 0.001$ kg/cc

$\therefore C_\Delta = 0.72$

for the model

$$\Delta (m) = C_\Delta \times wb^3 = 0.09 \text{ kg} = 90 \text{grms.}$$

Reynolds Number (based on the beam)

$$Rn = \frac{Ub}{V}$$

$$[Rn]\ s = \frac{7.0 \times 0.5}{1.00 \times 10^{-6}} = 3.48 \times 10^6$$

$$[Rn]\ m = \frac{2.0 \times 0.05}{1.007 \times 10^{-6}} = 9.93 \times 10^4$$

This is a significant difference in Reynold's number since the model is in the transition region from laminar to turbulent flow.

No turbulence inducing devices were used on the models because of the highly variable wetted length.

Lift co-effecient for the model

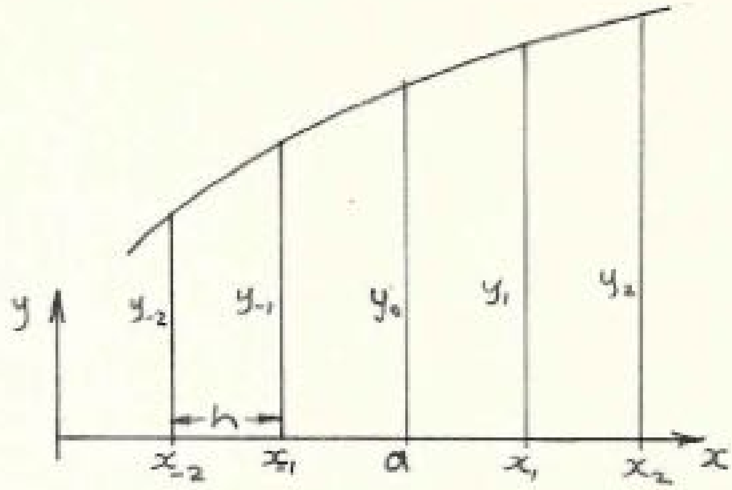
$$C_{Lb} = \frac{2C_\Delta}{Cv^2} = 0.18$$

The model dimensions are shown in figure 17 .

Wave profile measurement.

A simple profile measuring device was constructed from a block of wood, some brass welding rods and some spring steel wire. The dimensions of this device and measurements of a typical wave profile are given as an appendix to this thesis. The free surface angle was estimated from an approximate differentiation formula.

$$f'(a) = \frac{8(y_1 - y_{-1}) - (y_2 - y_{-2})}{12h}$$



Dimensions of the equipment

The following are some typical dimensions of the experimental equipment;

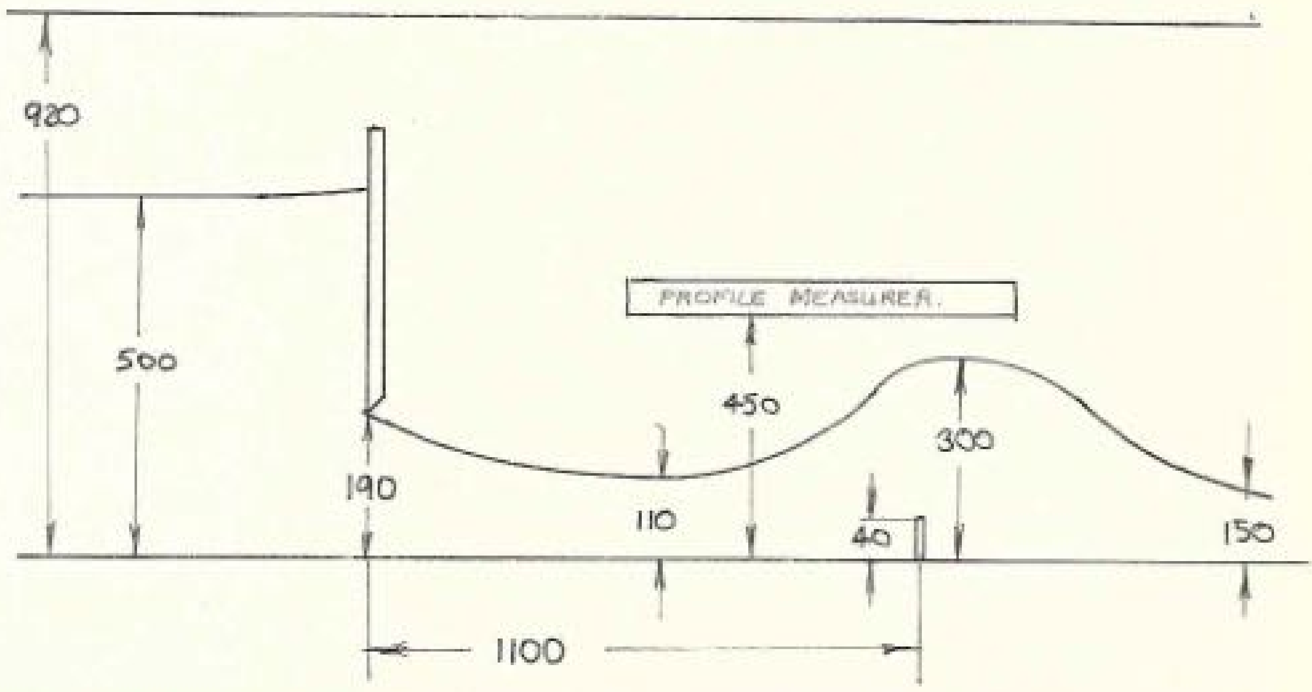
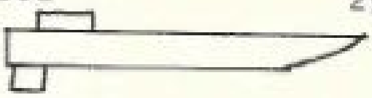


Figure 16.Dimensions of the equipment.(mm)

Model Dimensions.

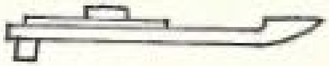
Model	length mm	beam mm	thickness mm	weight gms
1. Wood	290	80	20	590
2. Wood	200	55	6	140
3. Aluminium	170	51	2	160
4. Aluminium	170	51	2	160

1. Wood



Out of scale.

2. Wood



Adjustable load position

3. Aluminium

Point load, adjustable position

4. Aluminium

Point load, adjustable position
Single step as shown.

Towing balance dimensions. (mm)

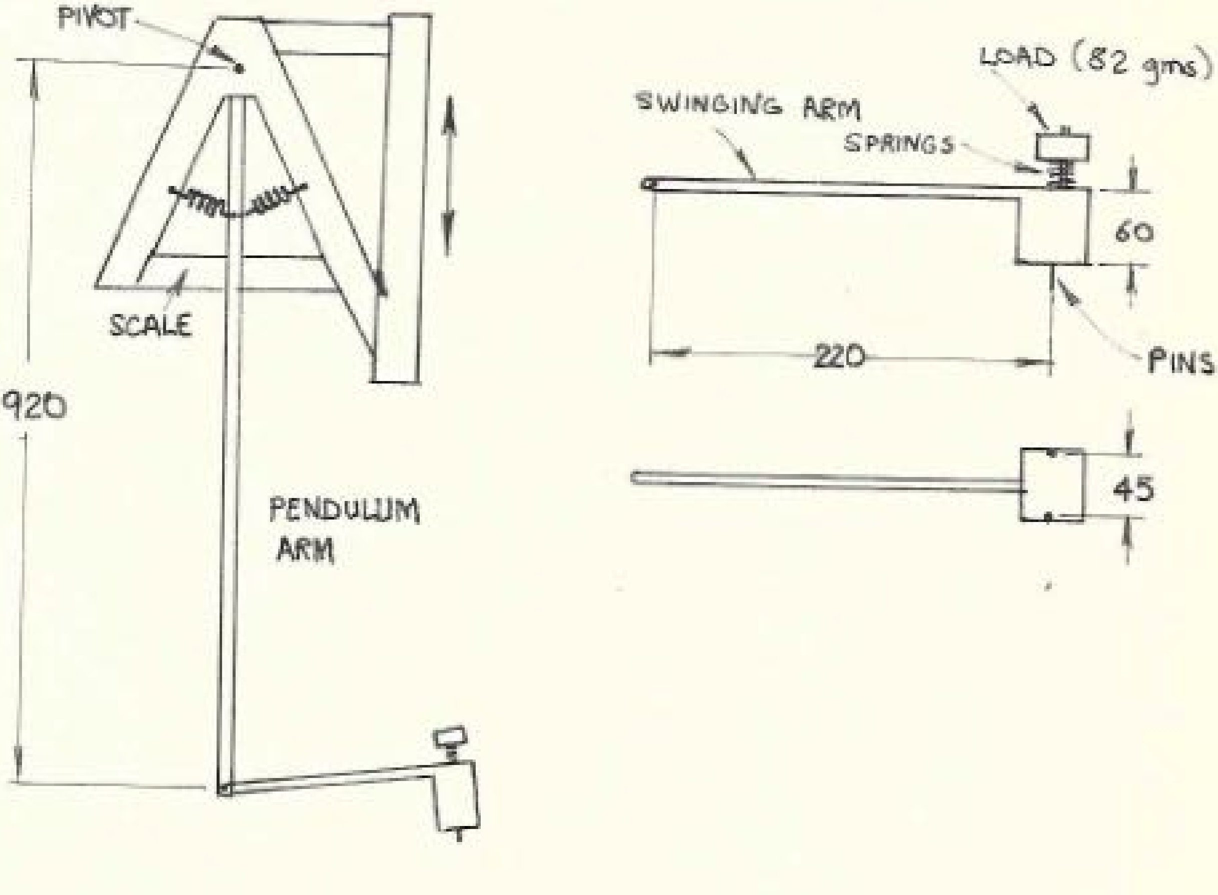


Figure 17. Dimensions of the towing balance.

Initial Results

Initialwork involved "playing" with model surfboards on the standing wave. This was quite productive since it indicated some problems and possible areas of investigation;

a) When placed on the wave, held only by string, the large model settled to an equilibrium position in which the string could be slackened. The model had two surfaces in contact with the water surface as shown. The phenomena was extraordinary in that it was stable in all modes. Sidewall effects apparently kept the model planing near the centre of the wave and the fin prevented the model from broaching. Also the model settled into a position on the wave where the lift/drag ratio was correct for equilibrium.

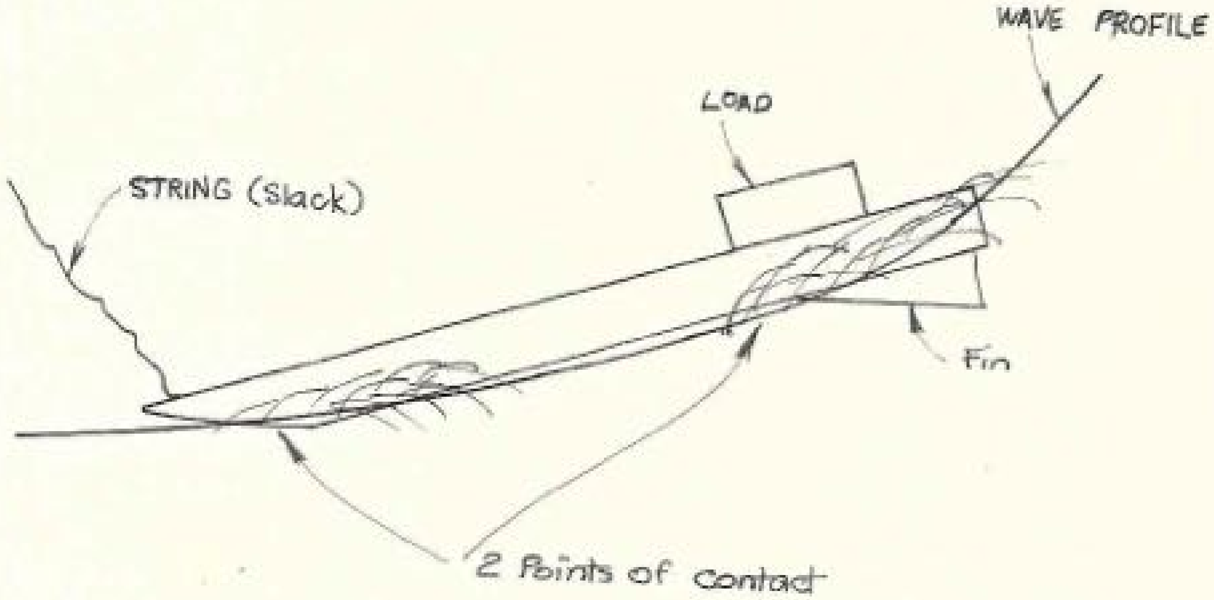
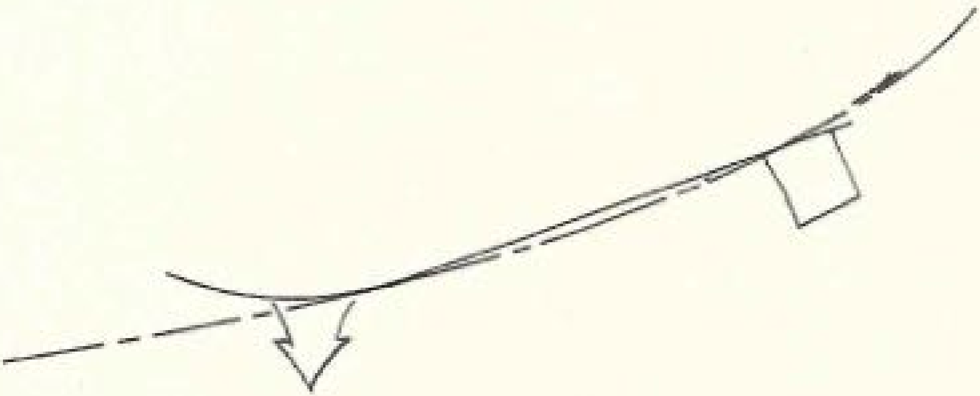
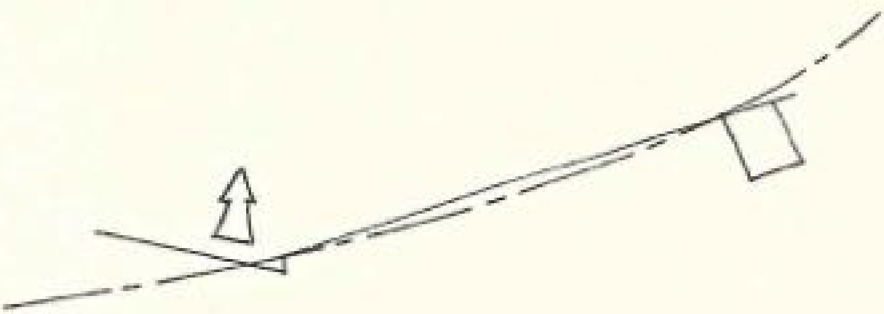


Figure 18.Equilibrium state of the model.

b) All the round-nosed models nose-dived at the bottom of the wave . Water flowing past the convex surface created a suction force which dragged the model underwater. This indicated that a stepped hull should be used to prevent nose-diving. A model with a single step was made and the model resisted attempts to sink it by the nose. This phemonema is shown in figure 19. .



Round nosed model is sucked under by the water flowing past the convex surface.



Flat nosed, stepped hull resists attempts to sink it by the nose.(Due to the pressures on the flat surface.)

Figure 19.Nose-diving of surfboards.

c) With the load applied at the stern, all models tended to porpoise when planing on a single surface. Porpoising is a coupled heaving and pitching motion, and is a problem in most stepless hulls. The porpoising was violent enough to break the main pivot pin of the balance, subsequently theoretical and practical investigation was made into the phenomena. The instability was not overcome before the end of term so none of the intended experiments (measuring equilibrium conditions) could be concluded.

Chapter 4 Stability of a Surfboard

Two types of instability are important in surfboard motion;

1) Direction instability of a flat surface, Broaching.
2) Porpoising.

The first is solved by adding a fin to the surfboard. The second is less simply overcome.

Porpoising

This is a coupled pitching and heaving oscillation, caused mainly by an oscillation of the centre of pressure. There may be several modes of the instability in different speed ranges. The factors affecting porpoising are more easily understood if a theoretical investigation is attempted. Several papers were published prior to World War II, covering porpoising theory but these need adaption for the surfboard case.

Theory

Consider a flat-bottomed planing craft with a centre of gravity in the plane of the board. The system is as shown.

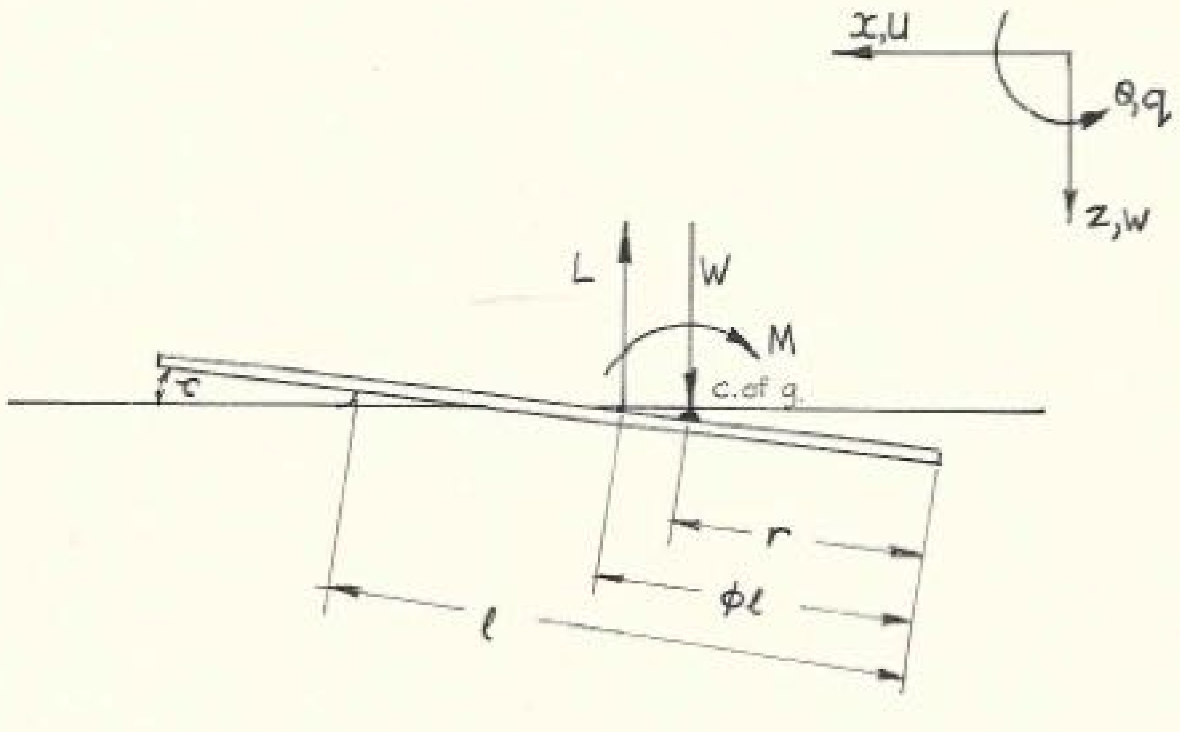


Figure 20. Porpoising notation.

If L is the vertical force, D is the horizontal force and M is the moment about the C. of g. then the equations of motion are;

$$m\ddot{x} - \dot{x}D_u - xD_x - wD_w - zD_z - qD_q - \theta D_\theta = 0$$
$$m\ddot{z} - \dot{x}L_u - xL_x - wL_w - zL_z - qL_q - \theta L_\theta = 0$$
$$mk^2\ddot{\theta} - \dot{x}M_u - xM_x - wM_w - zM_z - qM_q - \theta M_\theta = 0$$

Where $D_u = \frac{\partial D}{\partial u}$, $L_w = \frac{\partial L}{\partial w}$ etc.

Assume a solution of the form;

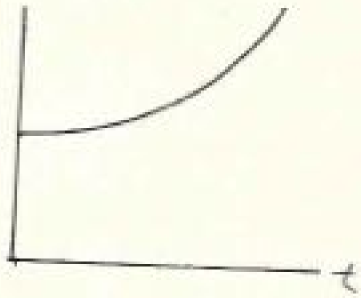
$$x = K_1 e^{\lambda t}$$
$$z = K_2 e^{\lambda t}$$
$$\theta = K_3 e^{\lambda t}$$

Where λ may be real or complex. The general stability equation is then;

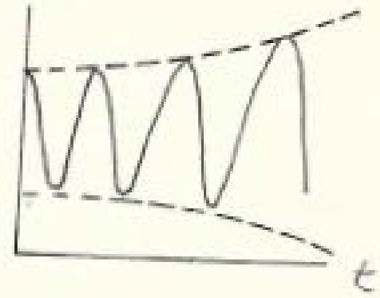
$$\begin{vmatrix} m\lambda^2 - \lambda D_u - D_x & -\lambda D_w - D_z & -\lambda D_q - D_\theta \\ -\lambda L_u - L_x & -m\lambda^2 - \lambda L_w - L_z & -\lambda L_q - L_\theta \\ -\lambda M_u - M_x & -\lambda M_w - M_z & mk^2\lambda^2 - \lambda M_q - M_\theta \end{vmatrix} = 0$$

This gives a 6th order polynomial in λ. There are four possible types of motion corresponding to the location of these roots on the Nyguist diagram.

UNSTABLE

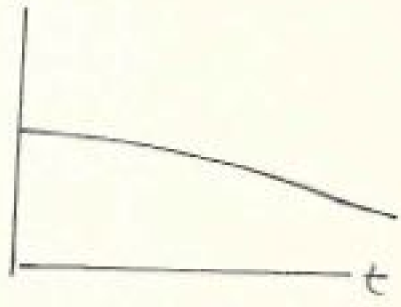


λ Real & +ve
Divergent

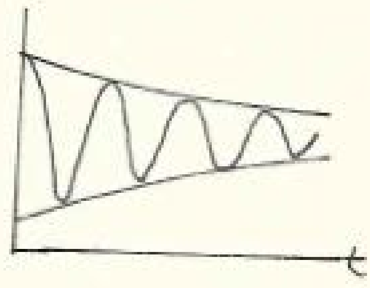


λ Complex, Real part +ve
Divergent Oscillation

STABLE



λ Real & −ve
Convergent



λ Complex, Real part −ve
Convergent Oscillation

Stability analysis on the equation is laborious and, because of some doubt on the x - derivatives, it was decided to consider a simplified system. Assume longitudinal motion has no effect on the stability. The stability equation becomes;

$$\begin{vmatrix} m\lambda^2 - \lambda L_w - L_z & -\lambda L_q - L_\theta \\ -\lambda M_w - M_z & mk^2\lambda^2 - \lambda M_q - M_\theta \end{vmatrix} = 0$$

Assume prior knowledge that $M_q = M_w = 0$ then;

$$\begin{vmatrix} m\lambda^2 - \lambda L_w - L_z & -\lambda L_q - L_\theta \\ -M_z & mk^2\lambda^2 - M_\theta \end{vmatrix} = 0$$

Therefore

$$m^2k^2\lambda^4 - (mk^2L_w)\lambda^3 - (M_\theta + k^2L_z)m\lambda^2 + (L_wM_\theta - L_qM_z)\lambda + (L_zM_\theta - L_\theta M_z) = 0$$

This is the characteristic equation of the system.

Now let $A = m^2k^2$

$$B = -mk^2L_w$$

$$C = -(M_\theta + k^2L_z)$$

$$D = (L_wM_\theta - L_qM_z)$$

$$E = (L_zM_\theta - L_\theta M_z)$$

Then Routh's discriminant is $R = D(BC-AD)-B^2E$. Routh's discriminant indirectly determines the sign of the real part of λ by testing the co-effecients of the characteristic equation. The necessary condition for stability is that

$R > 0$ and the critical case is for $R = 0$.

Therefore $$D(BC-AD) - B^2E = 0$$

Substituting for A, B, C.

$$D(m^2k^2L_w(M_\theta + k^2L_z) - m^2k^2D) - m^2k^4L_w^2E = 0$$

Collecting powers of k, the radius of gyration.

$$(DL_wL_z - L_w^2E)k^2 = (D - L_wM_\theta)D$$

$$\therefore \quad k^2 = \frac{(D-L_wM_\theta)D}{(DL_wL_z - L_w^2E)}$$

Dividing throughout by D

then $$k^2 = \frac{D - L_wM_\theta}{L_wL_z - L_w^2(E/D)}$$

but $D - L_wM_\theta = -L_qM_z$

$\therefore$ For stability

$$\boxed{k^2 \leqslant \frac{-L_qM_z}{L_w(L_w(E/D) - L_z)}}$$

From figure 20. $M = (\phi l - r)L$

$$\therefore \frac{\partial M}{\partial L} = \phi l - r$$

and $\frac{\partial M}{\partial l} = \phi L$

Vertical displacement derivatives L z, Mz.

Incremental displacement δz leads to an increase in wetted length;

For small τ

$$\frac{\partial l}{\partial z} = \frac{1}{\tau}$$

Let $\lambda = \frac{l}{b}$ then $\frac{\partial \lambda}{\partial l} = \frac{1}{b}$

therefore $\frac{\partial \lambda}{\partial z} = \frac{1}{\tau b}$

$$L_z = \frac{\partial L}{\partial z} = \frac{\partial \lambda}{\partial z}\frac{\partial L}{\partial \lambda} = \frac{1}{\tau b} L_\lambda \quad \ldots\ldots (1)$$

$$M_z = \frac{\partial M}{\partial z} = \frac{\partial M}{\partial L}\frac{\partial L}{\partial z} + \frac{\partial M}{\partial l}\frac{\partial l}{\partial z}$$

$$\therefore M_z = (\phi l - r)\frac{1}{\tau b} L_\lambda + \phi L/\tau \quad \ldots\ldots (2)$$

Angular (pitch) displacement derivatives L_θ, M_θ

$\delta\theta$ leads to a decrease in trim and an increase in wetted length; $(\delta\tau = -\delta\theta)$

$$L_\theta = \frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial l}\frac{\partial l}{\partial \theta} + \frac{\partial L}{\partial \theta}$$

but $\frac{\partial L}{\partial l} = \frac{L_\lambda}{b}$, $\frac{\partial l}{\partial \theta} = \frac{l-r}{\tau}$ $\left(\delta l = \frac{\delta\theta(l-r)}{\tau - \delta\theta}\right)$

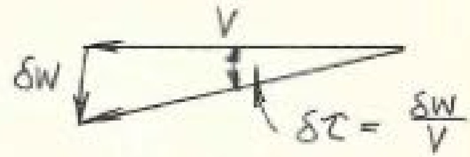
$$\therefore L_\theta = \left(\frac{l-r}{\tau b}\right) L_\lambda - L_\tau \quad \ldots\ldots (3)$$

$$M_\theta = \frac{\partial M}{\partial \theta} = \frac{\partial L}{\partial \theta}\frac{\partial M}{\partial L} + \frac{\partial l}{\partial \theta}\frac{\partial M}{\partial l}$$

$$\therefore M_\theta = (\phi l - r)(-L_\tau) + \left(\frac{l-r}{\tau}\right)\phi L \quad \ldots (4)$$

Vertical velocity derivatives Lw, Mw. Change in vertical velocity leads to an effective change in trim angle, but not wetted length.

$$\frac{\partial \tau}{\partial w} = \frac{1}{V}$$



$$L_w = \frac{\partial L}{\partial w} = \frac{\partial \tau}{\partial w}\frac{\partial L}{\partial \tau} = \frac{1}{V} L_\tau \quad \ldots\ldots (5)$$

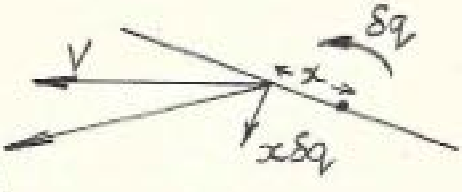
$$M_w = \frac{\partial M}{\partial w} = \frac{1}{V} M_\tau$$

but $M_\tau = (\phi \ell - r) L_\tau$

$$\therefore \; M_w = \left(\frac{\phi \ell - r}{V}\right) L_\tau \quad \ldots\ldots (6)$$

Angular Velocity derivatives Lq, Mq. Change in angular velocity leads to an effective change in trim given by:

$$\delta \tau = \frac{x\,\delta q}{V}$$



Consider the average change in trim along the board

$$\frac{\partial \tau}{\partial q} = \frac{1}{\ell}\int_{-r}^{\ell - r} \frac{x}{V}\,dx = \frac{1}{V}\left(\frac{\ell}{2} - r\right)$$

$$L_q = \frac{\partial L}{\partial q} = \frac{\partial L}{\partial \tau}\frac{\partial \tau}{\partial q} = \frac{1}{V}\left(\frac{\ell}{2} - r\right) L_\tau \quad \ldots\ldots (7)$$

$$M_q = \frac{\partial M}{\partial q} = \frac{\partial M}{\partial \tau}\frac{\partial \tau}{\partial q} = \frac{(\phi \ell - r)}{V}\left(\frac{\ell}{2} - r\right) L_\tau \quad \ldots\ldots (8)$$

Now for equilibrium oscillations $\phi l = r$ therefore the stability derivatives are;

$$L_z = \frac{1}{Tb} L_\lambda$$

$$M_z = \frac{\phi L}{\tau}$$

$$L_\theta = \left(\frac{l-r}{Tb}\right) L_\lambda - L_\tau$$

$$M_\theta = \left(\frac{l-r}{\tau}\right) \phi L$$

$$L_w = \frac{1}{V} L_\tau$$

$$M_w = 0$$

$$L_q = \frac{1}{V}\left(\frac{l}{2} - r\right) L_\tau$$

$$M_q = 0$$

The radius of gyration equation is;

$$k^2 \leq \frac{-L_q M_z}{L_w (L_w (E/D - L_z)}$$

Now $$-L_q M_z = \frac{1}{V}\left(r - \frac{l}{2}\right) \frac{\phi}{\tau} L L_\tau$$

$$L_w = \frac{1}{V} L_\tau$$

$$\therefore \quad \frac{-L_q M_z}{L_w} = \left(r - \frac{l}{2}\right) \frac{\phi}{\tau} L$$

$$E = L_z M_\theta - L_\theta M_z = \left(\frac{1}{\tau b}\right)\left(\frac{l-r}{\tau}\right) \phi L L_\lambda - \left(\frac{l-r}{Tb}\right) \phi L L_\lambda / \tau + \frac{\phi L L_\tau}{\tau}$$

$$\begin{aligned} D &= L_w M_\theta - L_q M_z \\ &= \frac{1}{V}\left(\frac{l-r}{\tau}\right) \phi L L_\tau + \frac{1}{V}\left(\frac{l}{2} - r\right) \frac{\phi L}{\tau} L_\tau \\ &= \frac{\phi L L_\tau}{V\tau}\left[\frac{3l}{2} - 2r\right] \end{aligned}$$

$$\therefore \quad \frac{E}{D} = \frac{V}{\left(\frac{3\ell}{2} - 2r\right)}$$

and $$L_W\left(\frac{E}{D}\right) = \frac{L_\tau}{\left(\frac{3\ell}{2} - 2r\right)}$$

$$\therefore \quad k^2 \leqslant \frac{\left(r - \frac{\ell}{2}\right)\phi L/\tau}{\left(\frac{L_\tau}{\left(\frac{3\ell}{2} - 2r\right)} - \frac{L_\lambda}{Tb}\right)}$$

Now for equilibrium oscillations $\phi \ell = r$

$$\therefore \quad k^2 \leqslant \frac{b\left(\phi - \frac{1}{2}\right)\ell\phi L}{\frac{TbL_\tau}{\left(\frac{3}{2} - 2\phi\right)\ell} - L_\lambda}$$

Normalizing this equation;

Let $\bar{k} = \frac{k}{b}$, $\lambda = \frac{\ell}{b}$, $L = C_L \frac{1}{2}\rho V^2 b^2$

then $$\bar{k}^2 \leqslant \frac{\left(\phi - \frac{1}{2}\right)\phi\lambda C_L}{\frac{\tau}{\lambda\left(\frac{3}{2} - 2\phi\right)}\frac{\partial C_L}{\partial \tau} - \frac{\partial C_L}{\partial \lambda}}$$

This is a non-dimensional equation for finding the limiting radius of gyration for stability.

Stability and $\bar{k}$, the nondimensional radius of gyration.

Recalling the equation on page 15;

$$C_L = \tau^{1.1}\left(0.012\sqrt{\lambda} + 0.0095\,\lambda^2/C_v^2\right)$$

The derivatives of this equation should be a good approximation for the stability derivatives because the function has a 'smooth' slope.

$$\frac{\partial C_L}{\partial \lambda} = \tau^{1.1}\left(0.006/\sqrt{\lambda} + 0.019\,\lambda/C_v^2\right)$$

$$\tau\frac{\partial C_L}{\partial \tau} = 1.1\,\tau^{1.1}\left(0.012\sqrt{\lambda} + 0.0095\,\lambda^2/C_v^2\right) = 1.1\,C_L$$

Therefore,

$$\bar{k}^2 \leq \frac{-\lambda(\phi - \frac{1}{2})\phi}{\dfrac{(0.006/\sqrt{\lambda} + 0.019\lambda/C_v^2)}{(0.012\sqrt{\lambda} + 0.0095\lambda^2/C_v^2)} - \dfrac{1.1}{\lambda(\frac{3}{2} - 2\phi)}}$$

Let $G = 6\sqrt{\lambda} + 19\,\lambda^2/C_v^2$

$$H = 12\sqrt{\lambda} + 9.5\,\lambda^2/C_v^2$$

Then
$$\bar{k}^2 \leq \frac{-(\phi - \frac{1}{2})\phi\lambda^2}{G/H - \dfrac{1.1}{\frac{3}{2} - 2\phi}}$$

Stability equation is;

$$\boxed{\bar{k}^2 \leqslant \frac{(\phi - \frac{1}{2})\phi\lambda^2}{\dfrac{1.1}{(\frac{3}{2} - 2\phi)} - \dfrac{G}{H}}}$$

Calculations for $\bar{k}$;

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
λ	$\tau°$	λ^2/C_V^2	G $6\sqrt{\lambda}+19(3)$	H $12\sqrt{\lambda}+4.5(3)$	G/H	$6.875-(6)$	$\bar{k}$ $\sqrt{\frac{.1139(2)}{(7)}}$
0.1	19.55	5.4×10^{-4}	1.91	3.80	.502	6.370	0.0134
0.4	10.34	8.65×10^{-3}	3.80	7.598	.500	6.375	0.053
1.0	6.62	0.054	7.03	12.50	.562	6.313	0.134
2.0	4.53	0.216	16.11	26.06	.618	6.257	0.270

These calculations indicate a linear relationship between $\bar{k}$ and λ: As a rule,

$$\bar{k} \leq 0.134\lambda \quad \text{for stability.}$$

Notes;

* At high trim angles the very low radius of gyration required for stability would be impossible to achieve. The structure would not be rigid, due to the light construction.

* the ratio G/H does not vary greatly over the relevant range of trim angles.

Discussion of Stability Theory

The importance of radius of gyration is indicated by this Theory, some other relevant points are;

a) The ratio of G/H does not vary greatly over the speed range of surfboards, therefore stability may be obtained by increasing the wetted length, for example by;

- Decreasing the trim, moving the load forward.
- Increasing the lift co-effecient by increasing the load or decreasing the velocity.

These are the usual suggestions given by authors for solving porpoising but in this case they defeat the purpose of the experiment, to model surfboard riding.

Since actual surfboards do not seem to porpoise some practical experiments were carried out on some possible damping phenomena. This involved investigation of human response to a moving platform.

The stability of an actual surfboard.

Theorectical and experimental results indicate that the surfboard - man combination should porpoise in the range of speeds encountered in surfboard riding. These results were based on symmetrically planing, the surfboard was travelling perpendicular to the wave crest. As mentioned previously this is not a common surf board riding case. Usually the surfer will skew the board to travel across the wave and apparently this attitude is more stable.

Occasionally however surfers do ride straight ahead and under these conditions porpoising theoretically should prove a nuisance. This does not appear to occur in practice. Some possible explanations are;

1) The waves were very small (4ft) and as a result the velocities achieved were low, the board may not have been truly planing. Therefore the conditions found in the laboratory were not representative of the case at the beach.

2) Human response may be significant in damping out the oscillations.

Human Response - The effect on stability

There are three responses that may aid stability;

a) Instinctive reaction to maintain the body centre of gravity over the feet. This effect is accompanied by moments applied by the feet during the reaction. Such a system was modelled by using a point loading (see experimental work), however the reaction moments were not modelled and this may be significant in the porpoising discrepancy.

b) Bending the knees to reduce vertical shock loads. This system is a complicated, variable spring - damper system which may have a significant effect in damping heave oscillations. This system has been modelled by placing the load on a low frequency spring system, however the frequency is critical and in the human case it can be highly variable so that accurate modelling is impossible.

c) Pitch oscillation damping by reactions through the ankles. Human responses also damp out pitching shock loads through the ankles. This is not as instinctive as the other two reactions but may be significant with surfboard riders since they learn precise control of pitch of the surfboard. This response has not been modelled in the experiments and may prove the critical response because the model has no pitch oscillation damping.

The attempts to model human response were not successful. The model still porpoised although the range of instability may have been decreased.

Another effect that may lead to the model instability is the effect of the curved free surface.

Effect of the curved free surface

It seems reasonable to assume that a curved water surface has some effect on porpoising so a comparison was made between the model planing on flat water and a curved water surface. The experiment was brief and simple but had a significant result.

The model surfboard was placed at the vena contracta of the sluice gate flow (Point of mimumum depth). There the water surface is horizontal, but concave. The load position was adjusted until porpoising commenced. For this case the critical point of load application was about 120 mm from the transom, well forward of midway.

The model and drag balance assembly was then attached to the main towing carriage and towed at 6ft/S, corresponding to the vena contracta velocity. Three locations of the load were tried. The system was stable with the load at 100 mm but porpoised with the load at 70 mm and 50 mm from the transom. There appears to be a decrease in stability with a concave water surface. This result may be worthy of further investigation.

Some other effects of the curved free surface may be;

1) A change in the pressure forces. The same pressure force causing the water to orbit will also act on the surfboard. Since the force is directed horizontal and forward at midheight, the additional pressure could be expected to increase the lift. Another way of explaining this effect is to use the analogy of the situation with a concave bottomed board. The effective increase in the change of momentum of water produces additional lift.

2) As mentioned previously the curved water surface increases thelikelyhood of nose-diving. The convex-nosed model nose-dived if any load was placed on the bow. However, this phenomena was not apparent on flat water. Nose-diving often has dangerous consequences and it was suggested that surfboards have stepped hulls to overcome the problem since even a roughly made stepped hull surfboard model resisted attempts to sink the nose.

These results may also be applicable to the design of surfboats and ocean racing craft.

Chapter 5 Conclusions

Effects of a waves on planing craft.

These can be summarized as follows;

a) It should be possible to "surf" deep water waves although there are several problems with this concept.

b) Model experiments may be applicable to surfboard design. The Theory developed here seems applicable but there are still some problems with direct application of towing tank data.

c) The range of stability is decreased. Porpoising is more likely on a wave then on flat water.

d) Round-nosed surfboards are prone to nose-diving. However this may be overcome by using a stepped hull. This phenomena may also be applicable to other ocean going planing craft.

Some surfboard design suggestions

Some of the more controversial areas of surfboard design such as the length and theshape of the "ideal" surfboard are avoided here. Instead simple suggestions are made, based on both experiment and research.

a) The edges where the water leaves the surfboard, such as the transom and sides at the tail should be as sharp as possible. Since very sharp edges may be dangerous a suggested profile for this region is shown in figure 21 .

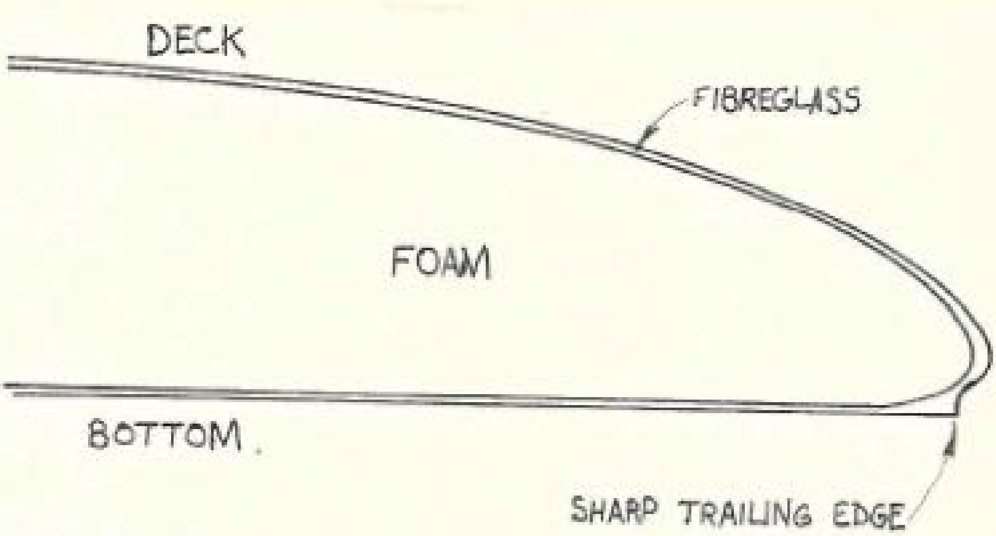


Figure 21. Design of the trailing edges.

b) The nose should be flat, not curved, with a small step as shown. This would prevent nose-diving, a dangerous phenomena that often deters beginners.

c) If a surfboard is difficult to turn and manoeuvre then a small skeg attached near half-length may increase manoeuvreability.

These suggestions are shown in figure22.

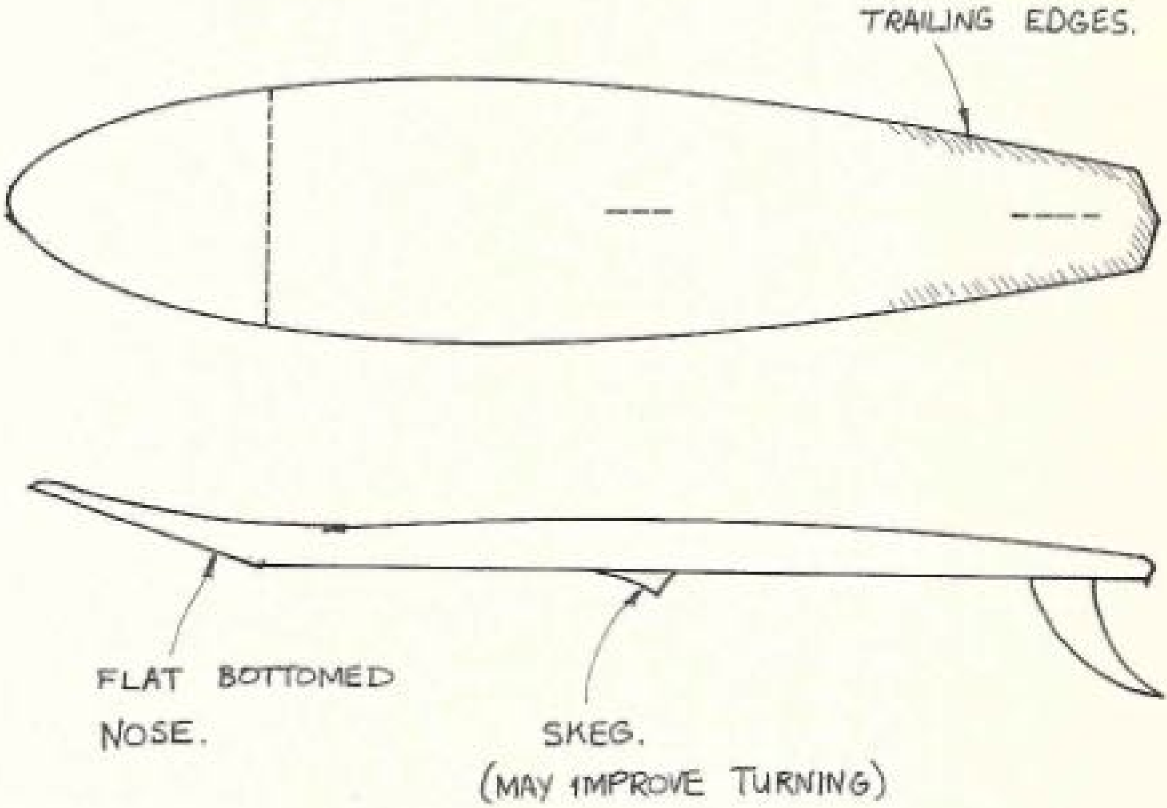


Figure 22.Suggested improvements to surfboards.

A concluding note

It would appear that the most important criterion in the performance of surfboards is the almost instinctive reaction by the rider as he senses changes in the relationship between the board and the wave. When this control system can be simulated experimentally another sport will have died!

References

The references have been divided into three sections. Wave Theory, Planing Theory and Experimental Work. Most are available in the P.N.R. library, the source of the exceptions is placed in brackets after the reference.

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$FORTRAN
C       CHARACTERISTICS OF DEEP WATER WAVES
C      BY M.P.PAINE...MECHANICAL ENG. 4
C    X,Y FREE SURFACE CO-ODS
C    ALPHA FREE SURFACE ANGLE
C    U,V PARTICLE VELOCITIES...W.R.T. WAVE...'R' IS THEIR RESULTANT, AT ANGL
C    DY  SAMPLE SPACING
C    K IS NO. OFSAMPLES +1
C    M IS THE NUMBER OF PROFILES OF VARYING AMPLITUDE
C    G  ACCL'N DUE TO GRAVITY
C    DB IS THE AMPLITUDE STEP.
      READ,WL,A,B,DY,J,K,M,G,DB
      S=6.284/WL
      B=1.125*S**2.0
      C=SQRT(G*WL/6.284)
      DO 88 I=1,M
      J=K
      YMAX=B/(1.0-1.72*B*S)
      YMIN=B/(.72*B*S-1.0)
      A=YMAX-YMIN            THIS IS ONLY AN APPROX. FOR THE WAVE AMPLITUDE.
      Y=YMAX
      WRITE(6,2000)WL,A,B,C
 2000 FORMAT(1H1,10X,24HTROCHOIDAL WAVE  PROFILE ,//,10X,15HWAVELENGTH(F
     *T)= ,F10.1,5X,14HAMPLITUDE(FT)= ,F10.1,/,10X,9HBETA(FT)= ,F10.1,
     *9X,20HWAVE VELOCITY(FT/S)=,F10.2,//,7X,2HX ,8X,2HY ,6X,5HALPHA,7X,
     *2HU ,8X,2HV ,8X,2HR ,6X,5HTHETA,5X,4HUDOT,6X,4HVDOT ,6X,4HRDOT,6X,
     *5HTHETR,5X,6HYCHECK )
   66 CONTINUE
      J=J+1
      IF(J.GT.100) GOTO 22
      YVAR=Y/(B*EXP(S*Y))
      IF(YVAR.GT.1.0) GOTO 44
      YNEG=0.0-YVAR
      IF(YNEG.GT.1.0) GOTO 22
      X= ARCOS(YVAR)*1.0/S
      YCHECK=B*EXP(S*Y)*COS(S*X)
      DYDX=S*EXP(S*Y)*SQRT(B*B-Y*Y/EXP(2.0*S*Y))/(S*Y-1.0)
      ALPHA=ATAN(DYDX)*57.3
      U=C*(1.0-B*S*EXP(S*Y)*COS(S*X))
      V=C*(0.0-B*S*EXP(S*Y)*SIN(S*X))
      R=SQRT(U*U+V*V)
      THETA=ATAN(V/U)*57.3
      UDOT=C*B*S*S*EXP(S*Y)*(U*SIN(S*X)-V*COS(S*X))
      VDOT=0.0-C*B*S*S*EXP(S*Y)*(U*COS(S*X)+V*SIN(S*X))
      RDOT=SQRT(UDOT**2  +VDOT**2  )
      THETR=ATAN(VDOT/UDOT)*57.3
      WRITE(6,3000)X,Y,ALPHA,U,V,R,THETA,UDOT,VDOT,RDOT,THETR,YCHECK
 3000 FORMAT(7(F10.2),3(F10.3))
   44 CONTINUE
      Y=Y-DY
      GOTO 66
   22 CONTINUE
      B=B-DB
   88 CONTINUE
      STOP
      END
$ENTRY

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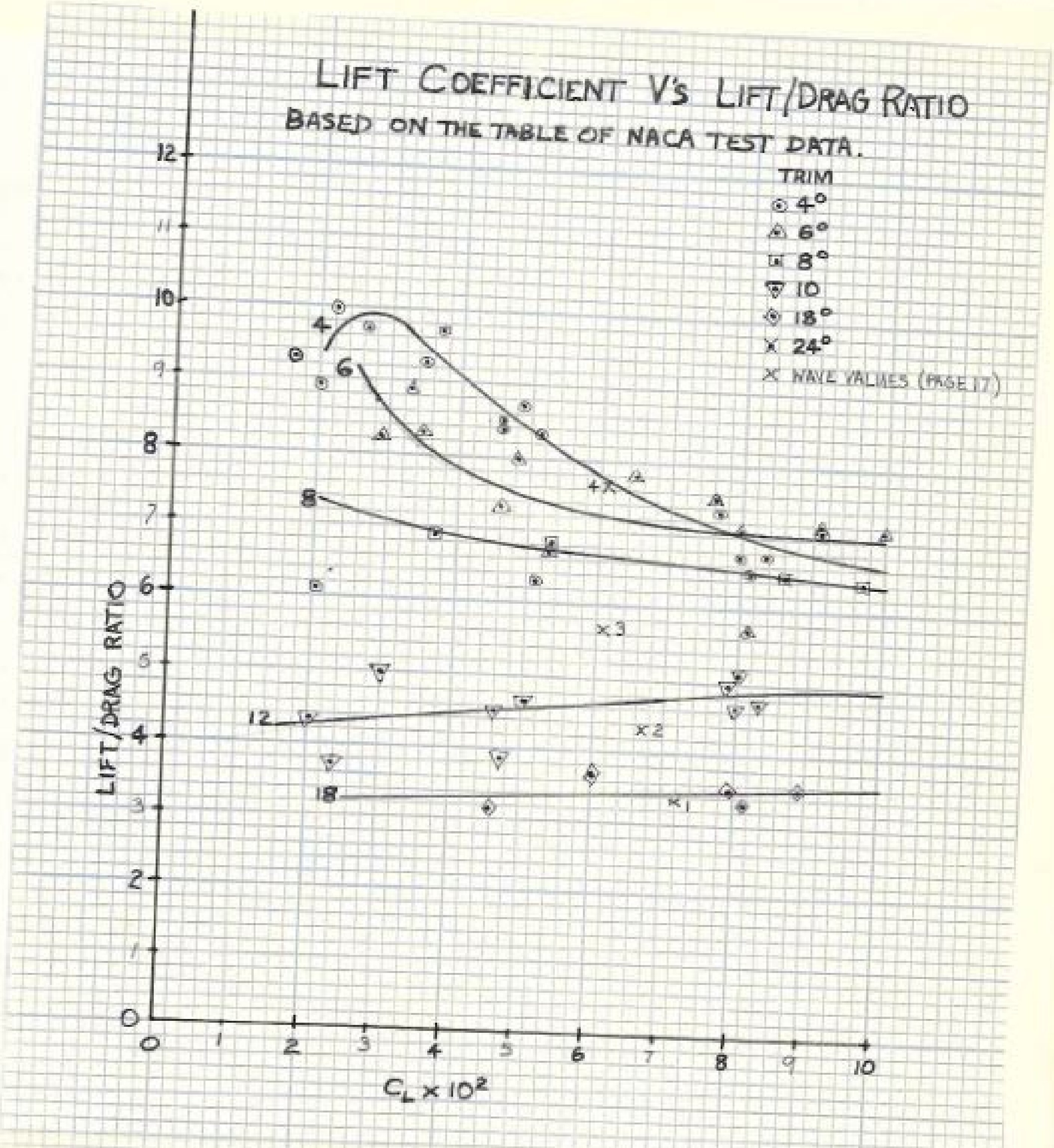
APPENDIX II Table of Lift/Drag Ratios, based on NACA model tests.

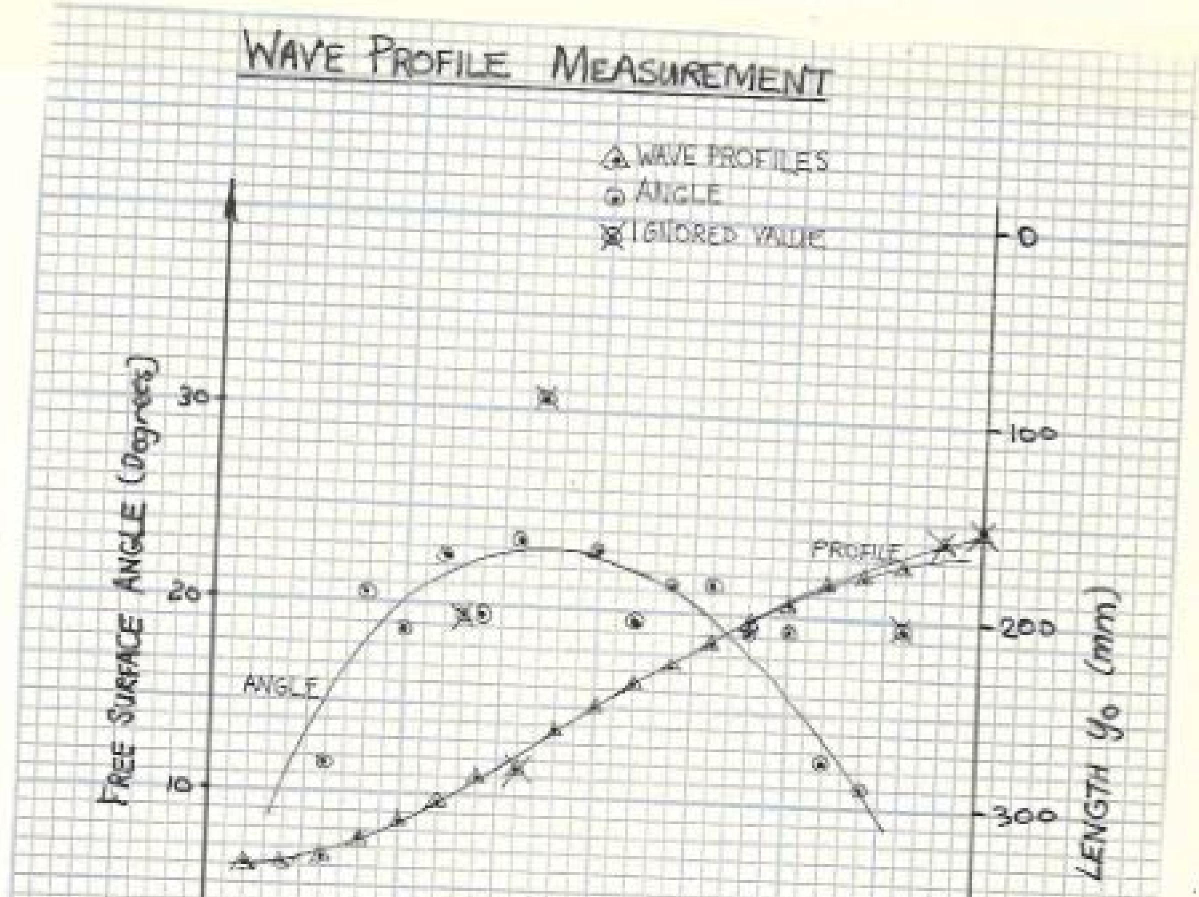
Trim τ	C_L	L/D
degrees	$\times 10^2$	–
Shoemaker Data (B.5)		
4	11.1	7.05
"	9.1	7.05
"	7.7	7.31
"	5.13	8.35
"	3.56	9.23
"	2.62	9.68
"	1.64	9.38
"	12.2	7.00
"	10.4	7.00
"	4.87	8.70
"	3.70	9.75
"	2.19	10.0
Weinstein Data (B.7)		
4	8.1	6.50
"	4.6	8.45
"	2.04	6.00
"	8.3	6.64
"	4.58	6.30
"	2.00	8.90
"	8.00	6.77
"	4.6	8.40
Shoemaker Data		
6	5.3	6.65
"	15.2	6.67
"	11.0	7.00
"	11.0	6.80
"	9.1	7.05
"	7.6	7.50
6	4.8	8.00
"	3.5	8.30
"	2.93	8.20
"	20.4	6.65
"	14.7	6.60
"	10.1	7.13
"	6.5	7.90
Weinstein Data		
6	8.11	5.68
"	4.6	7.36
"	18.2	5.60
"	12.8	6.68
"	3.23	8.96
"	8.00	7.27
"	12.1	6.65
Shoemaker Data		
8	20.0	5.8
"	18.0	5.9
"	15.2	5.85
"	12.8	6.15
"	9.7	6.40
"	8.6	6.45
"	5.3	6.80
"	3.7	6.90
Weinstein Data		
12	8.3	4.71
"	4.7	3.87
"	2.3	3.70
"	18.4	4.96
12	8.0	4.61
"	4.6	4.55
"	7.9	4.95
"	3.1	5.05
"	2.0	4.36
"	11.2	4.95
"	8.7	5.12
"	5.0	4.62
Weinstein Data		
18	8.1	3.27
"	4.6	3.13
"	18.3	3.06
"	12.5	3.38
"	7.9	3.50
"	8.9	3.55
"	6.0	3.71
Weinstein Data		
24	12.4	2.44
"	12.5	1.90
"	7.8	2.00
"	7.9	1.92
"	8.1	1.85

Shoemaker model had 16" beam.

Weinstein model had 4" beam.



LIFT COEFFICIENT V's LIFT/DRAG RATIO
BASED ON THE TABLE OF NACA TEST DATA.
TRIM
4°
6°
8°
10
18°
24°
WAVE VALUES (PAGE 17)
LIFT/DRAG RATIO
$C_L \times 10^2$
4
6
8
12
18
×1
×2
×3


WAVE PROFILE MEASUREMENT
WAVE PROFILES
ANGLE
IGNORED VALUE
FREE SURFACE ANGLE (Degrees)
30
20
10
ANGLE
PROFILE
0
100
200
300
LENGTH y_0 (mm)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
STATION (30mm Spacing)

Appendix III. Wave Profile Measurement.

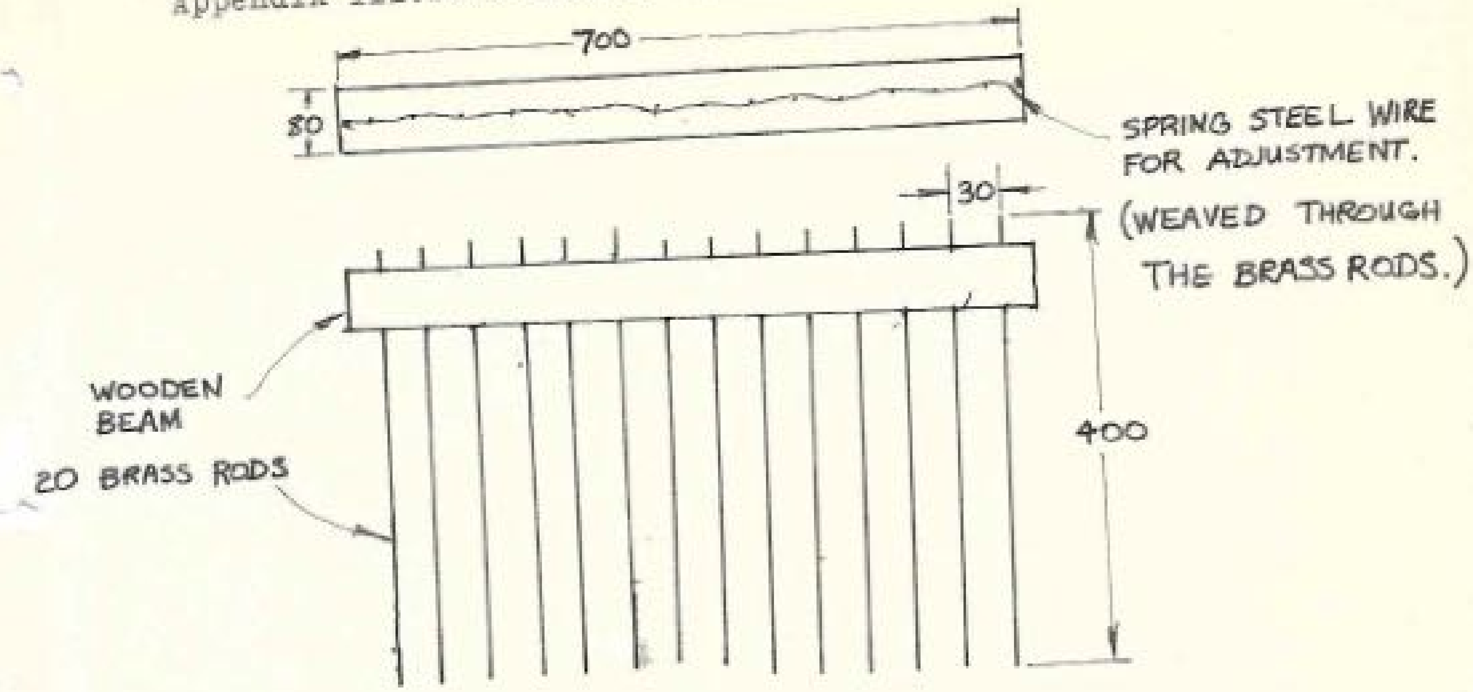


Table of calculations. (Page 37)

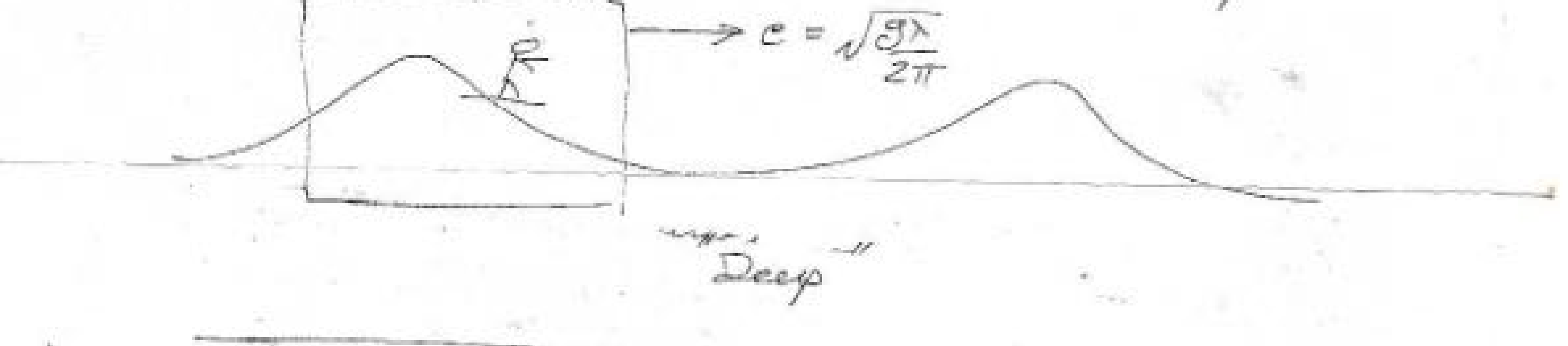
STATION (1)	y_o (2) mm	$y_1 - y_{-1}$ (3)	$y_2 - y_{-2}$ (4)	$\frac{8(3) - (4)}{360}$ (5)	$\tan^{-1}(5)$ (6)	ESTIMATED ANGLE (see Graph) (7)
20	153					
19	160					
18	172	20	32	·360	19·6	8·0
17	180	13	34	·194	11·0	11·0
16	185	14	33	·220	12·4	13·5
15	194	20	35	·347	19·1	16·0
14	205	21	43	·347	19·1	18·0
13	215	23	44	·389	21·2	20·0
12	228	23	45	·386	21·1	21·0
11	238	22	49	·353	19·4	22·0
10	250	26	55	·425	23·0	22·5
9	264	33	52	·569	30·5	23·0
8	283	26	54	·427	23·2	23·0
7	290	21	40	·357	19·6	22·5
6	304	24	42	·416	22·6	21·5
5	314	21	45	·341	18·9	20·0
4	325	21	34	·372	20·4	18·0
3	335	13	36	·188	10·7	15·0
2	338					11·0
1	340					

Thesis: Hydrodynamics of a Surfboard with particular reference to a board travelling at an angle to a wave crest θ less than 90°.

Computer Program

(A)(i) By way of gaining some sense of magnitude consider a cycloidal wave in deep water, say $\lambda = 300$ feet. Find velocity & acceleration components for all particles at water surface.

✓ (ii) Try to make the unsteady flow into a steady flow by a Gallilean transformation



✓ (iii) Consider the statics of the situation. Consider the minimum lift/drag ratio required to sustain planing action.

✓ (IV) Can this lift drag ratio be achieved? See Barnaby: What beam at transom would be required? NACA Data

Hydrodynamics
Lamb

[signature]
20 March, 1974